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Robustness Analysis

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http://www.casos.cs.cmu.edu/

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Contents

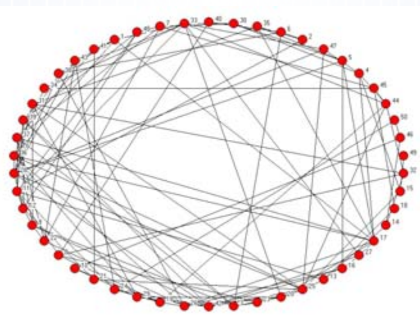
- Network Topology Basics
- ORA Activity – Generate Random Networks
- Robustness Of Centrality Measures

Network Topology

- Uniform Random
- Small-World
- Scale-Free
- Core-Periphery
- Cellular

Uniform Random

Spring Embedded Layout



Circular Layout

Existence of a tie is a function of a fixed probability.

50 nodes, 10% density, undirected

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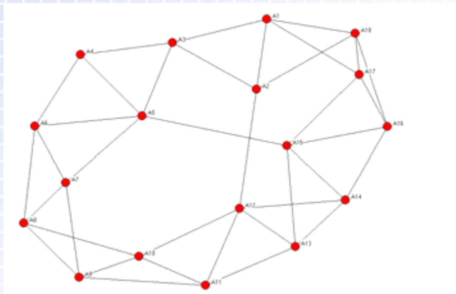
Small-World



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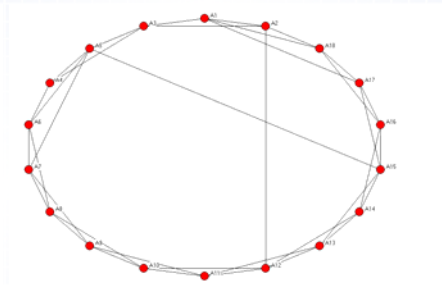

Circular Layout

Nodes are tied to n-immediate neighbors; then a random few are re-wired to other nodes, selected at random.

50 nodes, 10% density, undirected

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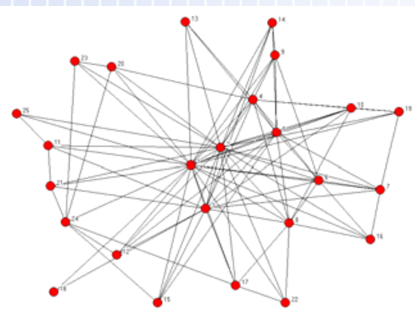
Scale-Free



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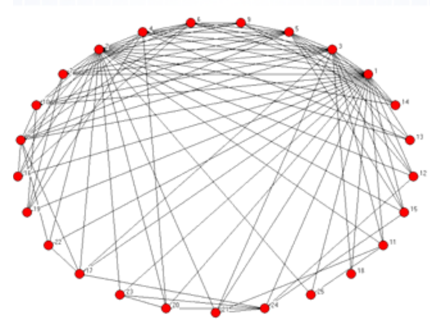

Circular Layout

Network is grown; preferential-attachment for ties to others with high degree centrality.

50 nodes, 10% density, undirected

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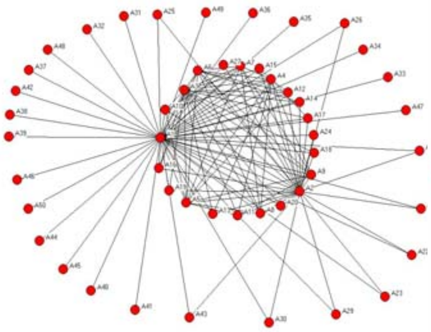
Core-Periphery



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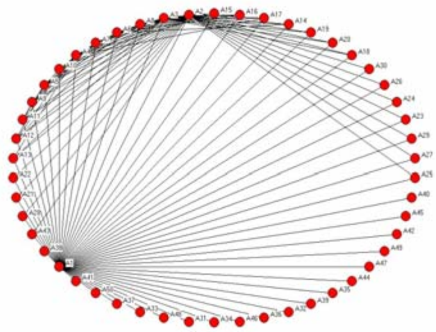

Circular Layout

Nodes in the core subset are highly connected to one another; nodes in periphery connected to few in the core.

50 nodes, 10% density, undirected

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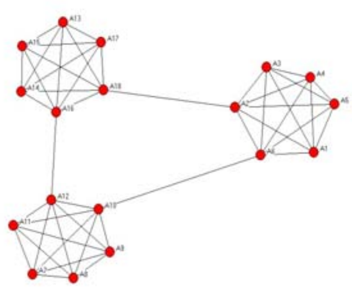
Cellular



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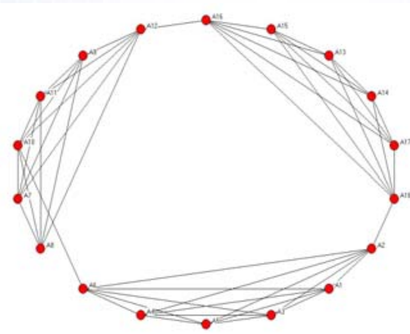

Circular Layout

Cell subgroups are highly connected within; cells are loosely connected.

50 nodes, 10% density, undirected

Activity

Task:

Conduct a comparative analysis of network measures for a pair of topologies.

Process:

Working in a group, use ORA to conduct a social network analysis comparing an example <topology-of-your-choice> network with an example random network.

ROBUSTNESS EXPERIMENTS

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Problem

Social-network data is prone to error…

Documented nodes and ties may be quite mistaken

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Motivation

- Explore the robustness of social network measures
- Given that missing and inaccurate data is the norm, then how reliable are the measures?
- What caveats should we assign to interpreting the measures?
- Can the bounds and robustness of measures be quantified under conditions of data error?

Borgatti, Carley, & Krackhardt (2005)

Findings:

- Measure accuracy declines predictably with increasing error
- Four centrality measures have similar robustness pattern and levels
- Type of the error (node / edge) has little affect on the robustness
- Increased density reduces accuracy for all kinds of error, except edge addition where accuracy increased.

Our Research Question

How is network topology related to the robustness of measures of centrality?

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Methodology

- Run O(2,400) controlled experiments under varied scenarios to generate data. In each experiment:
  - Select sample networks O(10,000) randomly from a bounded population of candidates (TRUE NETWORK - *T*)
  - Independently perturb each sample (OBSERVED NETWORK - *O*)
  - Collect the respective measures for each *T*, *O pair.*
- Compute descriptive statistics of measures each experiment
- Aggregate data from independent experiments into various configurations for analysis

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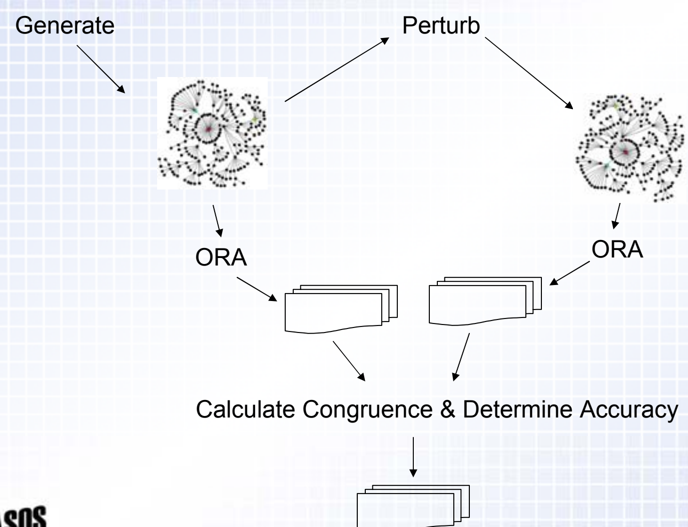
Experiments



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Factorial Design

- Each experiment/scenario* has 5 independent attributes:
  - Topology (3) (Uniform Random, Cellular, Core-periphery)
  - Node Size(4): 10, 25, 50, 100
  - Density(8): 1%, 2%, 5%, 10%, 30%, 50%, 70%, 90%
  - Introduced Error:
    - Type(5): EdgeRemove, EdgeAdd, NodeRemove, NodeAdd, NodeAlias
    - Level(5): 1%, 5%, 10%, 25%, 50%
- Factorial Design Experiment
  - 3x4x8x5x5 = 2,400 experiments
- * Except for topology, experiment replicates Borgatti, Carley, & Krackhardt (forthcoming).

Experiment Design

- Each experiment trial (*n*=2,400x10,000) has 4 graph-level-measure responses:
  - Betweenness Centrality
  - Closeness Centrality
  - Degree Centrality
  - Eigenvector Centrality

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Experiment Design

- Each of the 7 graph-level-measure responses is represented by 6 criteria:
  - Top 1 – Proportion Rank 1 in True is Rank 1 in Observed
  - Top 3 – Proportion Rank 1 in True is Rank 1-3 in Observed
  - Top 10% - Proportion Rank 1 in True is Rank Top 10% in Observed
  - Intersection R-Squared – Pearson correlation between T & O
  - Union R-Squared – Pearson correlation between T & O
  - Overlap – (Num nodes in T top 10% and O top 10%) / Num in either 10%
    - i.e., Overlap = |NT ∩ NO | / | NT U NO |
- Each criteria is reflected in 4 statistics:
  - Minimum
  - Maximum
  - Mean
  - Standard Deviation

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Experiment Design

For example, each experiment (*n*=2,400) results in 7 of these unique charts:

Uniform Topology; 100 Nodes; 50% Density; Edge Remove; 10% error (*n*=10,000)

Betweenness Centrality

	Minimum	Maximum	Mean	Std. Dev.
Top 1	0.000	1.000	0.484	0.500
Top 3	0.000	1.000	0.795	0.404
Top 10%	0.000	1.000	0.983	0.129
Intersection R-Squared	0.804	0.953	0.899	0.020
Union R-Squared	0.804	0.953	0.899	0.020
Overlap	0.111	1.000	0.529	0.135

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Current Experiments

Run 3 full experiments (Uniform, Cellular, Core-periphery)

- 33.6 million graphs produced
- 840,000 data values to analyze

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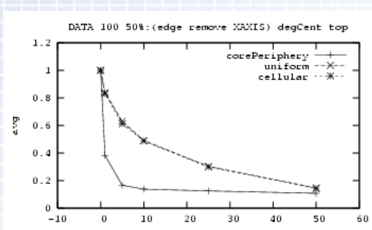
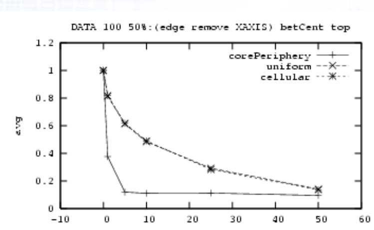
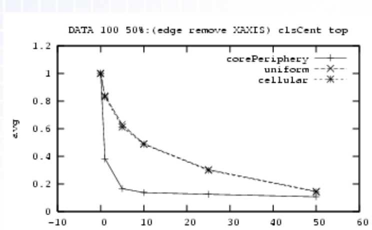
Four Centrality Measures Similar



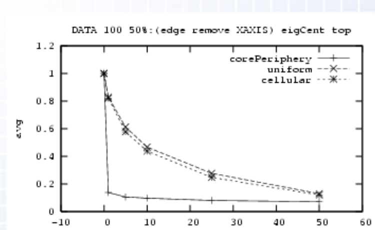
DEGREE



BETWEENNESS



CLOSENESS



EIGENVECTOR

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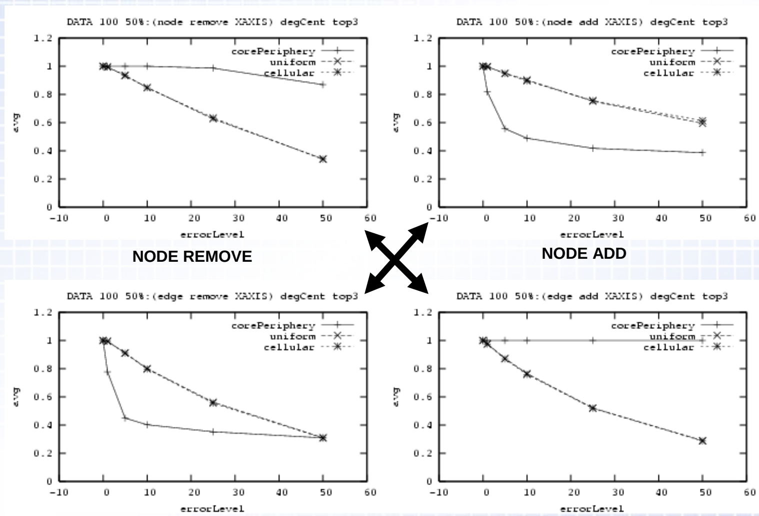
Uniform vs. Core-Periphery



NODE REMOVE

NODE ADD

EDGE REMOVE

EDGE REMOVE

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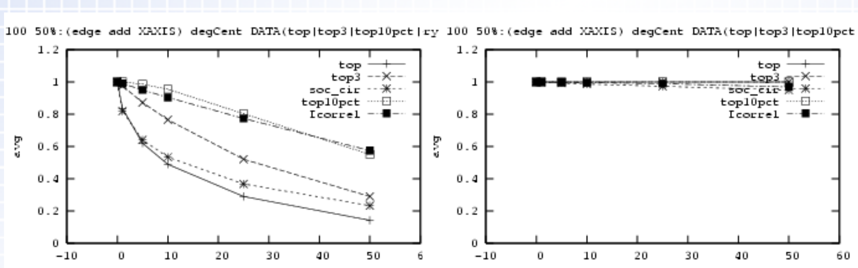
Uniform vs. Core-Periphery



UNIFORM

CORE PERIPHERY

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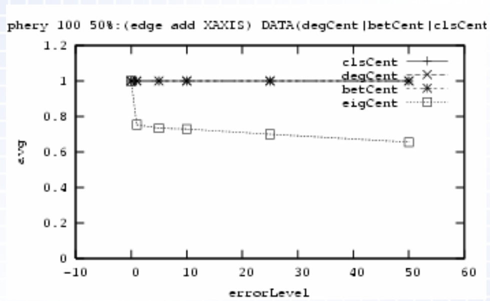
Core Periphery



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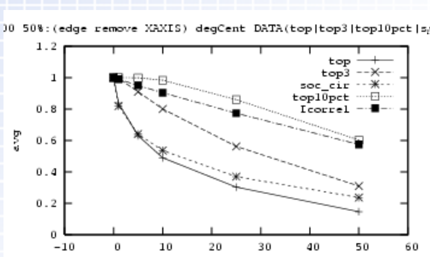
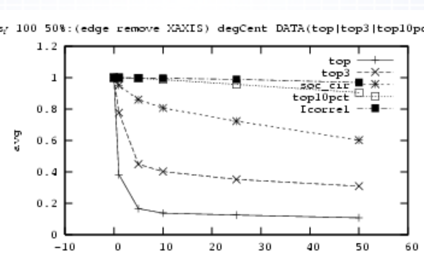
Uniform vs. Core-Periphery



UNIFORM



CORE PERIPHERY

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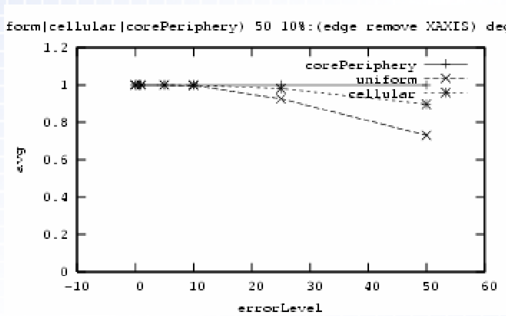
Cellular



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Findings

1. Accuracy profile for all four centrality measures is similar
2. Core-Periphery to uniform/cellular is more robust in case of node-remove and edge-add; less robust in cases of node-add and edge-remove.
3. Core-Periphery edge-add has very high (near 1) robustness across all error levels
4. Core-Periphery less robust to smaller levels of error.

