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Assessing the Political Landscape: Structure, Cognition, and Power in Organizations

David Krackhardt
Cornell University

This paper argues that an accurate cognition of informal networks can itself be a base of power, above and beyond power attributable to informal and formal structural positions. To explore this claim, a small entrepreneurial firm was studied. Perceptions of the friendship and advice networks were compared to "actual" networks. Those who had more accurate cognitions of the advice network were rated as more powerful by others in the organization, although accuracy of the friendship network was not related to reputational power.•

The Bush team apparently failed to understand that Sen. Tower, though he had served in the Senate for 24 years, didn't leave behind a reservoir of good feeling. "I don't think they understood how despised John Tower was by his peers while he was there," says Democratic consultant Mark Siegel. [From an article analyzing why Bush failed to win confirmation of John Tower as secretary of defense, *Wall Street Journal,* March 10, 1989]

In his seminal book on power, Pfeffer (1981) included a chapter on how to assess who has power in organizations. He argued that this is an important question not only for researchers but also for political actors in organizations. That is, it is not enough to have power; one should also know the political lay of the land—how much and what kinds of power others have. Pfeffer quoted Pettigrew (1973: 240) at the beginning of this chapter: "An accurate perception of the power distribution in the social arena in which he lives is . . . a necessary prerequisite for the man seeking powerful support for his demands."

But how does one assess the political landscape in an organization? One way of addressing this question is to identify the key political actors in the organization, a task about which Pfeffer had much to say. But simply identifying the most powerful actors may not give sufficient information to anticipate the overall dynamics of resistance and support for political acts. Additional questions about these actors come to mind: Are these powerful actors organized such that they tend to act in unison? Do they represent different political constituencies? Precisely whom does each have influence over? Beyond knowing who is powerful, it is useful to know how the powerful and powerless are organized, or structured. As Bailey (1969: 108) noted: "The man who correctly understands how a particular structure works can prevent it from working or make it work differently with much less effort than a man who does not know these things."

One way to approach the answers to these deeper questions about the political landscape is to study access to and the control of information flow in the organization (Pettigrew, 1973). As far back as 1965, Hubbell derived both a measure of the power of individual actors and an identification of powerful coalitions, using the actors' networks of ties. Laumann and Pappi (1976) documented how power accrued to those in central network positions in a community of elites. Brass (1984) discovered that centrality in work-related communication networks was a robust predictor of power in a printing company. As Pfeffer (1981: 130) stated: "Clearly, the power that comes from information control . . . derives largely from

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one's position in both the formal and informal communication networks."

More to the point, the study reported in this paper suggests that power accrues not only to those who occupy central network positions in organizations but also to those who have an accurate perception of the network in which they are embedded. An individual who has an astute knowledge of where the network links are can have a substantial advantage. First, this information provides a good assessment of who is powerful in the organization, since the central actors in the network can be easily ascertained. Knowing who the central—and powerful—actors are in the organization is essential political knowledge. Second, this information can be used to identify where the coalitions are in an organization. Knowing where the coalitions are, how large they are, and where their support comes from gives one an edge in anticipating resistance and in mobilizing support for action or change. Third, an accurate assessment of the network can also reveal the weaknesses in political groups by exposing holes, gaps, and locations of lack of support for any particular coalition. Thus, understanding the network provides a source of power independent of centrality in the network.

The central point in this paper is precisely that: Cognitive accuracy of the informal network is, in and of itself, a base of power. Both the concepts of power and cognitive accuracy are further developed below. In addition, I will argue that these two concepts are embedded in a structural context that must be taken into account in any empirical exploration.

Power

There has been much disagreement as to the precise meaning of power. Some writers have referred to it as the ability to get things done despite the will and resistance of others, the ability to "win" political fights, or a capacity to outmaneuver the opposition (Bierstadt, 1950; Emerson, 1962). Others (e.g., McClelland, 1975; Kanter, 1979; Roberts, 1986) have stressed the positive sum nature of power, suggesting it is the raw ability to mobilize resources to accomplish some end (without specific reference to organized opposition). Still others refer to power as the ability to control premises of actions, such that power becomes almost unobservable (Bachrach and Baratz, 1962; Lukes, 1974; Mizruchi, 1983). Salancik and Pfeffer (1977) preferred to ignore these distinctions, noting that, while academics may quibble over the definition of power, those actually experiencing the effects of power in the real world seem to exhibit a consensus as to who has it.

Without fully resolving this debate, it is reasonable to assume that the answer to the question of who has power depends in part on an answer to the question, Power to do what? If the influence being sought is within the routine operation of the organization, then people who are "experts," people in "authority," and, generally, people who know how things work around the organization are likely to be seen as powerful. If, however, the influence entails a radical departure from prior operations, then the uncertainty that emerges is likely to arouse emotional responses to influence attempts. Affect-laden issues such as trust, respect, or liking may become

important in evaluating who has the ability to mobilize support for the radical change (Krackhardt and Stern, 1988). In such cases, the powerful person may be someone who has referrent power (French and Raven, 1968) or charisma (House, 1977; Bradley, 1987; Fiedler and House, 1988) in the organization rather than someone who simply has authority or expertise.

Following the advice of Pfeffer (1981), I have included multiple kinds of power in this study. The assumption is that some actors are powerful because they are acknowledged as adept at getting things done in the organization, despite some resistance (e.g., Brass, 1984) and that some actors are influential because of an ascribed individual trait that reflects intangible qualities of trust and personal charm. These two different assessments of power are offered as ones that actors will readily recognize as influence bases in organizations: the ability to get things done in spite of resistance and the ability to influence people through personal appeal and magnetism (which is termed charisma).

Cognitive Social Structure and Accuracy

Cognitive accuracy, the major independent variable in this study of power, requires the development and measure of two subsidiary concepts. First, one must identify a standard or "actual" network against which accuracy can be measured.[1] Second, one must assess each individual's cognitive representation of this standard. Accuracy, then, is simply how well the individual's cognitive map approximates the standard.

Both the "actual" and cognitive maps of the network can be derived from what has been called cognitive social structures (Krackhardt, 1987). Networks on any given relation have been traditionally represented by a square matrix, $R_{i,j}$, where i is the sender of a relation and j is the receiver of the relation. Krackhardt argued that such relations might be fruitfully represented by three dimensions $R_{i,j,k}$, where k is the perceiver of the relation from i to j. There are several implications of this framework for looking at networks. For one, relationships are often based on people's perceptions and interpretations and not necessarily on observable, behavioral fact. For example, if two people act cordially toward each other, but underneath that cordiality they despise each other, it does not make sense to call them friends even though they may be observed to act like friends. Another implication of this three-dimensional framework is that the perceptions of the relationship from i to j can vary substantially. Person i may have a different view of the i-to-j relationship than person j has; both of these views may differ from that of a third bystander, person k. Krackhardt (1987: 119–125) found that such perceptions do vary considerably.

The current study was motivated by the question: How closely does each person's perception of the network approximate the "actual" network and how does this relate to power? To address this question, two types of aggregations were employed: The set of N individual perceived maps of the whole network, called "slices" of $R_{i,j,k}$; and the "actual" network, as defined by the two people actually involved in the relationship, referred to as the locally aggregated structure, or LAS (Krackhardt, 1987).

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I use the term "actual" in quotes here and throughout this paper to qualify its familiar meaning. The relationships referred to here may include behavioral components, but these behaviors in and of themselves do not define the relationships. Thus, "actual" networks are not defined by behaviors, per se, between actors, and therefore are not directly observable. Rather, the existence of an actual relationship is defined consensually by the two actors engaged in the relationship.

Just as power itself is a multidimensional concept, network relationships may be assessed on several dimensions. But the specific question is, what network relations are critical for the assessment of power? For example, a network composed of incidental communication links, such as perfunctory "Hello's," may not be as rich in power information as a network composed of critical advice relationships. The study reported here was based on the cognitive social structures for two different types of networks that have been shown to be useful in understanding the dynamics of informal organizations (e.g., Lincoln and Miller, 1979; Burt, 1982: 25; Brass, 1984; Krackhardt and Porter, 1985, 1986). First, the advice network (who goes to whom for work-related advice) represents the instrumental, workflow-based network in the organization. The second network assessed was the friendship network, or what Lincoln and Miller (1979: 186) called the "primary network." This latter network is not necessarily linked to the routine work done in the organization, but it does capture important affective and social bonds that can affect trust, especially in times of change (Krackhardt and Stern, 1988).

Structural Influences

In pursuing issues of power, one cannot ignore critical contextual and structural factors that also operate to give certain actors privilege and power in an organization. Brass (1984) found that centrality in the informal network itself predicts power. But centrality also has important theoretical links to cognition (see Krackhardt, 1987, for a comparison of different types of centrality). A series of studies has found that central involvement in a social system increases one's ability to "see" the social system accurately (Freeman and Romney, 1987; Freeman, Romney, and Freeman, 1987). Freeman, Freeman, and Michaelson (1988) noted that "social intelligence," the ability to discern social groups and boundaries, evolves over time as participants gain experience in the social group. Freeman and Romney (1987) demonstrated that people's ability to recall social structure accurately was a function of whether they were members of the core group or were peripheral, transitory members. These results, combined with Brass's (1984) findings, suggest that centrality in the informal structure can lead to both cognitive accuracy and power.

Another structural power base that cannot be ignored is the formal position a person has in the organization. Clearly, those with more authority will have more power, on the average, than those with less authority. In addition, those higher in the organizational chart are responsible for a larger part of the organization. A first-line supervisor is responsible for the activities of his or her immediate subordinates. A manager of several supervisors is responsible for these supervisors and ultimately for the activities of all their subordinates. People's positions require them to pay attention to the way in which those under them work together and relate to each other. Thus, those higher up in the organization will have, by virtue of their position, a better opportunity to observe and take note of a larger part of the informal network. Consequently, they are likely to have a more accurate picture of the informal

network. This should be particularly true in a small, entrepreneurial firm, where the owners/managers are known to be heavily involved in the details and day-to-day workings of the entire organization.

Those higher in the formal organization are forced to relate to a wider base of people. A first-line supervisor must coordinate the activities of a limited number of people, all of whom are likely to interact informally with each other and be doing similar work. A top-level manager must coordinate the activities of supervisors and managers from different functions and sectors of the organization. This responsibility gives higher-level managers more central positions in the formal organization, in that they will find themselves dealing with more issues that surface between departments or groups. This formal role is likely, in turn, to lead to opportunities to be in the middle of the informal network, acting as a bridge between groups of employees. Therefore, it is expected that formal hierachical level will also contribute to network centrality.

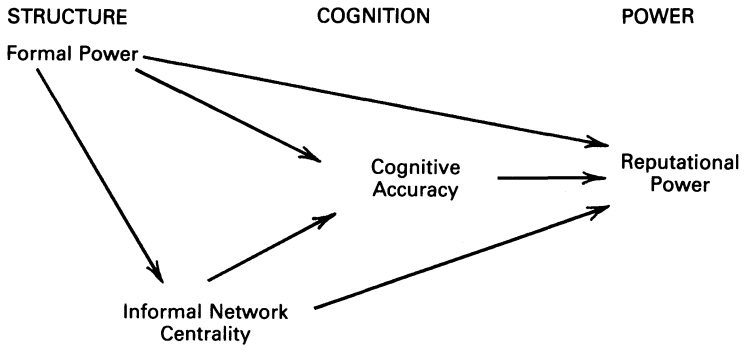
There are thus both structural and cognitive power bases in an organization. While it is proposed here that cognitive accuracy is a power base in and of itself, one must take into account the fact that this cognitive power base is influenced by formal and informal structural factors. Since these structural factors are sources of power in their own right, these sources are explicitly included as part of the cognitive model of power presented here.

Figure 1 displays this model, which relates structure, cognition, and power. Formal structure is shown as an exogenous variable leading directly to informal structure, cognitive accuracy, and power. Informal structure, in turn, contributes to cognitive accuracy and power. Finally, in accordance with the central theme of this paper, cognitive accuracy is predicted to contribute to power over and above the power already explained by the structural factors. This last link represents the main proposition of this paper:

Proposition: Controlling for formal and informal bases of power, cognitive accuracy of the informal network will be correlated with individual power in the organization.

To test the model in Figure 1 and the proposition posed above, a network study was conducted of a small hi-tech firm. Questionnaire and interview data were collected from

Figure 1. Model relating structure, cognition, and power.



which the cognitive social structures and "actual" networks were determined, and each employee provided assessments of how powerful and charismatic every employee was in the organization. From these data, the central proposition and model were tested directly.

METHOD

The Site

A small, entrepreneurial firm, called here Silicon Systems, was selected for study. The firm was located in an area known for its many small, start-up firms as well as some more established ones. Silicon Systems' business involved the sales, installation, and maintenance of state-of-the-art information systems in client organizations. Their clients ranged from local banks to schools to medium-sized manufacturing firms to R&D labs. Until recently, their largest competitors, such as IBM and AT&T, had focused their marketing efforts on the neighboring metropolitan areas. But, of late, these competitors were beginning to pay more attention to Silicon Systems' market because of the growth potential of that market. According to the top managers of Silicon Systems, the small firm's competitive edge rested in its ability to respond more efficiently to idiosyncratic customer demands.

Silicon Systems was wholly owned by the three top managers, each of whom owned an equal share. All employees worked in the single-floored building owned by the company. They saw each other regularly, although the installers spent many days at sites rather than in the office. Thus, employees were familiar with each other to varying degrees, and each had an opinion about every other employee, with the occasional exception of new hires.

The firm had grown from three people to 36 people in fifteen years. Much of this growth occurred in the five years previous to the study. Most of these years had been profitable, and the owners anticipated no downward trend in their business.

Reputational Measure of Power

A reputational measure of power was developed following the work of Brass (1984). In his study of nonsupervisory personnel, he obtained ratings from each person's supervisor and nominations of "who is influential" from the nonsupervisors. Such estimates have bias problems, since ratings are not based on comparable sources. One supervisor may provide higher overall ratings than another, even though the latter has more powerful subordinates. And, as Brass noted, nominations of colleagues who are "influential" have potential availability-bias problems stemming from the fact that respondents are likely to list only people who are currently salient in their minds. Moreover, it provides a dichotomous value that hides people's views of the degree of power they ascribe to their colleagues. The limitations of these measures were unavoidable because of the size of Brass's sample ($N = 160$). Despite these limitations, however, Brass demonstrated reasonable internal consistency using different sources and different methods, and his reputational measures

correlated well with his nonreputational measure of power, future promotion.

Because Silicon Systems was so small, the question of reputational power could be addressed more directly by asking each employee to rate all the employees (including him- or herself) on the two dimensions of power: the ability to get things done despite resistance and the ability to influence through personal magnetism (charisma). This procedure avoided the problem of availability bias, incomparable sources, and dichotomization. Moreover, with this multiple-source method, the internal reliability of the two power scores can be estimated. The directions for this part of the questionnaire operationally defined the two measures for the respondent:

> There are many ways in which people influence each other in the workplace. We are interested in two of these: charisma and power. For purposes of this questionnaire, we will define charisma as the influence derived from personal magnetism. A person who is charismatic tends to generate loyalty and enthusiasm in others. Power, on the other hand, is the ability to get things done that the power-holder wants done, in spite of whatever resistance he or she may encounter. People who are powerful can get most of what they want.

Each person rated each other person on a 7-point Likert scale on both charisma and the ability to get things done (potency). Two anchors were provided for each scale: "Not at all charismatic" to "Highly charismatic" for charisma, and "Not at all powerful" to "Highly powerful" for potency.

Formal Position

While power derived from formal position may be ambiguous in some larger organizations, this organizational base of power was quite clear in Silicon Systems. There were three distinct levels of formal authority. At the top level were the three owner-managers. Even though they took on different responsibilities and had different titles, they were equal partners and made all major company decisions jointly. The next level consisted of five managers, each of whom had supervisory responsibility over certain operational features in the organization. The remaining 28 employees had no formal supervisory title or authority. Formal position, then, was scored as follows: each of the three owners was given a formal position score of 3; the five managers were given a formal position score of 2; the remaining 28 employees were given a formal position score of 1.

Cognitive Accuracy

The cognitive social structure is a three-dimensional array of linkages, $R_{i,j,k}$, among a set of N actors, where i is the sender of the relation, j is the receiver of the relation, and k is the perceiver of the relation. Using Krackhardt's (1987) methodology, a questionnaire was designed to assess the cognitive social structure of two relations in the organization, friendship and advice. The directions for the "advice" section of the questionnaire were as follows:

> In this section, you will find a set of similar questions with a list of people after each question. The question is: "Who would this person go to for help or advice at work?" That is, if this person had a question or ran into a problem at work, who would they likely go to to ask for advice or help? Please answer the question by placing

a check next to the names of all the people the person is likely to go to. . . . Some people may go to several people for help or advice. Some may only go to one person. Some may not go to anyone, in which case do not check anyone's name under that question.

These directions were followed by 36 questions (e.g., "Who would Cindy Stalwart go to for help or advice at work?"), each asking the same question about a different employee. Each of these 36 questions was followed by a list of 35 names, any of which the respondent could check off in response to the question.

Similarly, another section of the questionnaire asked about friendship. The directions for this section paralleled those in the previous section and are reproduced here in part:

This time the question is: "Who would this person consider to be a personal friend?" Please place a check next to all the names of those people who that person would consider to be a friend of theirs.

Again, the question was repeated 36 times, once for each employee's name (e.g., "Who would Cindy Stalwart consider to be a personal friend?"), and each question was followed by a list of 35 names, any number of which the respondent could check off.

Two different aggregations of these cognitive social structures provided the basis for the two independent variables, "actual" network and "perceived" network. Each is specifically defined below.

Actual network. While recent work in the area of recall of network relations has cast doubt on an informant's ability to relate accurately to whom they actually talk on any given day (see Bernard et al., 1984, for a review), Freeman, Romney, and Freeman (1987) have shown that people are remarkably good at recounting enduring patterns of relations that they have with others. Thus, while people may not remember whom they talked to today or this week, they can accurately tell you whom they are in the habit of relating to over an extended period of time. Consistent with these results, Brass (1984) found that the workflow network in his study closely corresponded to the network reported by respondents. Since it is these enduring relational patterns that are of interest—as evidenced by the wording in the questions—the locally aggregated structure, or LAS (Krackhardt, 1987), was used as a proxy for the "actual" network. The LAS is an aggregation defined by the local participants in the network. It mimics the typical form in which network data are collected. Technically, the definition is as follows for the "actual" advice and friendship networks:

$$R^*_{i,j} = \begin{cases} 1 \text{ if } R_{i,j,i} = 1 \text{ and } R_{i,j,j} = 1; \\ 0 \text{ otherwise.} \end{cases}$$

Both i and j must agree that i goes to j for help and advice before the $i \rightarrow j$ link is recorded as existing in the "actual" advice network.[2] Similarly, both i and j must agree that i considers j a friend before the $i \rightarrow j$ link is recorded as existing in the "actual" friendship network. Since the relationship is defined as existing when both parties agree that it exists, this measure of the "actual" network is direct and has obvious face validity.

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Since three of the 36 employees chose not to fill out the questionnaire, the following decision rules were used to deal with missing values. If either $R_{i,j,i}$ or $R_{i,j,j}$ but not both, were missing, then $R^*_{i,j}$ was equal to the nonmissing value. If both were missing, then a voting scheme was used to determine the presence of a tie: $R^*_{i,j} = 1$ if $\frac{1}{n}\sum_{k=1}^{n} R_{i,j,k} \geq .5$ for all nonmissing k; else, $R^*_{i,j} = 0$.

Cognitive accuracy. Each participant's cognition of the network was taken simply from the set of responses he or she selected on the network questionnaire. These responses are technically defined for person k as:[3]

$$R_{k_{i,j}} = \begin{cases} 1 \text{ if } R_{i,j,k} = 1 \text{ and } k \neq i \text{ and } k \neq j; \\ 0 \text{ if } R_{i,j,k} = 0 \text{ and } k \neq i \text{ and } k \neq j \end{cases}$$

There are several measures that can be used to assess the degree to which a person's perceived links correspond to the actual links. In a general form, the following 2 × 2 table portrays all the relevant information:

$R_{k_{ij}}$ \ $R^*_{i,j}$	0	1
0	a	b
1	c	d

One could use simple percentages here to assess accuracy, such as percentage of links correctly identified [$d/(d + b)$]. But such simple percentages do not take into account other types of accuracy. For example, one could just as easily argue that the percentage of nonlinks correctly identified as nonlinks [$a/(a + c)$] is an important component of accuracy, or perhaps the percentage of identified links that are correct [$d/(d + c)$] or the percentage of identified nonlinks that are correct [$a/(a + b)$].

There exists a family of measures of correspondence for such 2 × 2 tables, measures that account for the information contained in all four of the above percentages. Fifteen such measures were reviewed and analyzed for desirable properties by Gower and Legendre (1986). Of these measures, one (called S_{14} in their article) stood out by exhibiting high resolution (appropriate sensitivity to small changes in correspondence) and low nonlinearity (low distortion at extreme values). This correspondence measure is defined as follows:[4]

$$S_{14} = \frac{ad - bc}{\sqrt{(a + c)(b + d)(a + b)(c + d)}}$$

Using simple arithmetic, this formula can be rewritten to show how this statistic combines the four percentages referred to earlier:

$$S_{14} = \sqrt{\left(\frac{a}{a + c} - \frac{b}{b + d}\right)\left(\frac{a}{a + b} - \frac{c}{c + d}\right)}$$

Given its statistically superior properties and appropriateness in this case, the S_{14} statistic was used to measure the degree of correspondence between an individual's map of the network and the "actual" network. Henceforth, I refer to this measure as the accuracy score for individual k, since it is how accurate the individual is in recounting the "actual" network that is of theoretical interest here. ADVACCUR and

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This aggregation contains only perceptions of ties (and nonties) between dyads in which ego (k) is not involved. The local ties (defined as those where $k = i$ or $k = j$) are excluded from this set of perceived ties because to include them would confound the measures of cognitive accuracy and the actual network, which is based on local ties only. In fact, this exclusion does not alter the subsequent results substantially. Only a small fraction of observations (2/36, or less than 6 percent) is eliminated. The correlations between accuracy scores that include local ties and scores that exclude local ties are .96 and .95 for advice and friendship, respectively. Nonetheless, to avoid even the small degree of confounding that does occur, these local ties were excluded in the analysis reported here.

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This statistic has also been called the point correlation coefficient and is equal to the value obtained by computing a simple Pearson correlation coefficient between the original elements in $R^*_{i,j}$ and $R_{k_{i,j}}$.

FRNACCUR are the variable names given to the accuracy scores for each respondent's cognitive map of the advice and friendship network, respectively.

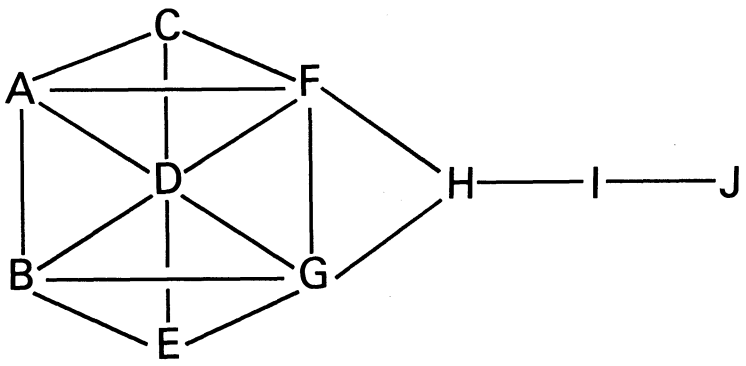
Centrality

The graph-theoretic concept of centrality has a long and distinguished history in the area of network analysis (Bavelas, 1948; Stephenson and Zelen, 1989). The three most common measures of centrality—degree, closeness, and betweenness—are compared and reviewed by Freeman (1979).[5]

Of the three indices, betweenness is the one most closely aligned with the idea of power; as Freeman (1979) put it, the individual who is in between other actors has more control over information flow from one sector of the network to another. That person becomes a gatekeeper of information flow. Moreover, betweenness is an indication of the nonredundancy of the source of information. To the extent that a person is connected to different parts of the network, and therefore has access to different, nonredundant sources of information, that person will have a wider variety of information at his or her disposal.

All three of these measures can be illustrated by the position of actors in the hypothetical network in Figure 2. I call this network a "kite structure," and it represents the smallest network I have found in which the centrality based on each of the three measures reveals different actors as the most central in the network. Degree centrality is defined as the number of links connected to the person. For example, D has the most links, with a degree centrality of 6. Closeness centrality is defined as the inverse of the average path distance between the actor and all others in the network. Persons F and G have the highest closeness centralities in the network.

Figure 2. Network exhibiting a kite structure.



Betweenness is somewhat more complicated in its definition. Using Freeman's (1979) notation, betweenness centrality is defined as follows:

$$C_B(k) = 2\sum_{i}^{n}\sum_{j}^{n}\left[\frac{g_{ij}(k)}{g_{ij}}\right]$$

for all unordered triples $i,j,k (i < j$, and $i \neq j \neq k)$, where n is the number of nodes in the network, g_{ij} is the number of geo-

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Another class of measures in network analysis, most strongly associated with Hubbell (1965), Bonacich (1987), and Salancik (1986), also carries the name "centrality." But the measures developed by these authors are aimed more at the concept of asymmetric status hierarchy, or "being at the top," than they are at the idea of "being at the center," which is the idea behind the graph-theoretic measures used here.

desics (shortest paths) between nodes i and j in the network, and $g_{ij}(k)$ is the number of geodesics from i to j that include k. If one thinks of a network as a roadmap, and one starts at point i and travels to point j, the question is does one travel through point k to get there? To the extent that k lies on the shortest paths between each pair (i,j), then k would be said to have high betweenness centrality. In Figure 2, H has the highest betweenness centrality. H, in essence, can block or control communication from one end of the network to the other. H also has access to parts of the network that are not connected to each other, allowing richer, more differentiated information to reach H. D, F, and G are "better connected" (have more connections) than H, but their connections are connected to each other, so that the information contained within these various parts tends to be redundant.

Centrality is expected to lead to power because it provides an exclusive control of information flow. In comparing the three centrality measures, Freeman (1979: 221) argued that a "person in such a position [such that he or she has a high betweenness score] can influence the group by withholding or distorting information in transmission." As such, betweenness captures the essence of what I mean theoretically by the concept of centrality. Moreover, since betweenness suggests access to differentiated quarters in the network, it is reasonable to argue that a person with the best betweenness position in the network stands to have a broader, more accurate overall picture of the network. Consequently, I used Freeman's betweenness index to measure centrality in this study.

Asymmetric relations. Freeman's review of graph-theoretic measures of centrality is based on symmetric graphs: the relationships represented in the network are assumed to be reciprocated. For example, if person A communicates with person B, then it is assumed that person B communicates with person A. Relations in this study, however, were not inherently symmetrical. Simply because A goes to B for advice does not mean that B goes to A for advice. It is even possible that friendships might be nonsymmetrical. One can imagine a situation in which A considers B a personal friend but B does not consider A a personal friend.

There are two ways to treat these asymmetries. On the one hand, one could assume that information only travels in the one direction specified by the asymmetric relation. For example, if A goes to B for advice, then one could assume that relevant information flow is from B to A and not vice versa. On the other hand, it may be more reasonable to assume that information flows in both directions as the result of an exchange, independent of who initiates the exchange. That is, just because A defers to B (by going to B for advice) does not mean that no information is passed from A to B in the exchange. In fact, by the very act of asking for advice, A is providing B with information about what A is doing or what is going on around A. For this reason, I assumed, in this study, that the presence of a relationship from A to B indicates an opportunity for an exchange of information in both directions, from A to B and from B to A. To operationalize this assumption, the measurement of centrality of an actor in the network was based on a symmetrized form of R^* defined as follows:

$$R^*_{S_{i,j}} = \begin{cases} 1 \text{ if } R^*_{i,j} = 1 \text{ or } R^*_{j,i} = 1; \\ 0 \text{ otherwise.} \end{cases}$$

Advice centrality and friendship centrality are the variable names representing the betweenness centrality of each actor in the "actual" advice and friendship network, respectively.

Table 1 contains the raw data for the analysis performed in this study. The two parts of the power variable are given separately. Data are not included for persons 13, 24, and 35 because they did not fill out the network questionnaire and were therefore excluded from the analysis. The ID number is the number I assigned to the respondent for identification. The raw data on the cognitive social structure choices for the advice network are provided in the Appendix.

Table 1

Power, Positional, and Accuracy Scores

ID	Potency	Charisma	Formal position	Advice centrality	Friend centrality	Advice accuracy	Friend accuracy
1	2.193	3.000	1	0.82	0.00	0.481	0.386
2	2.843	5.281	1	0.14	64.85	0.332	0.344
3	2.750	3.625	1	0.00	10.66	0.373	0.390
4	3.612	4.838	1	0.00	31.04	0.425	0.331
5	6.187	3.843	2	119.77	0.00	0.485	0.292
6	5.625	4.875	2	37.18	53.43	0.459	0.334
7	3.531	4.468	1	1.15	0.70	0.422	0.310
8	2.354	2.258	1	0.40	0.00	0.371	0.155
9	2.871	4.906	1	0.00	1.59	0.441	0.398
10	3.437	4.656	1	9.48	0.00	0.332	0.199
11	2.531	3.968	1	0.00	16.72	0.429	0.383
12	1.718	2.218	1	14.54	5.61	0.211	0.329
14	2.580	3.516	1	0.00	14.90	0.313	0.187
15	2.766	3.566	1	2.79	1.42	0.429	0.301
16	6.225	5.290	3	19.38	41.72	0.419	0.323
17	5.322	4.906	2	34.22	0.00	0.514	0.340
18	4.483	4.468	1	4.75	1.66	0.435	0.332
19	6.875	4.875	3	97.61	62.30	0.410	0.349
20	3.687	4.593	1	4.96	39.57	0.378	0.362
21	3.586	4.482	1	8.16	54.15	0.413	0.355
22	3.093	4.218	1	2.23	0.00	0.352	0.275
23	1.687	2.500	1	0.14	0.00	0.416	0.352
25	2.645	3.354	1	33.02	0.00	0.390	0.177
26	3.031	5.000	1	0.43	4.13	0.474	0.414
27	5.031	4.187	2	19.14	1.36	0.397	0.336
28	2.812	4.281	1	0.00	0.00	0.499	0.291
29	5.062	5.625	1	31.31	96.39	0.384	0.361
30	4.125	4.281	2	32.65	8.19	0.430	0.441
31	3.200	3.892	1	0.00	0.00	0.422	0.380
32	1.741	2.225	1	0.00	0.00	0.378	0.325
33	2.906	4.625	1	1.63	39.35	0.301	0.381
34	2.937	4.187	1	0.00	11.63	0.469	0.406
36	6.069	4.800	3	0.66	0.00	0.417	0.220

RESULTS

Reliability of Dependent Variables

It was assumed that people's reputation of power in the organization was an attribute of the individual whose reputation was being assessed. That is, since the firm was small, it was assumed everyone had a reasonably consistent and reliable view of how much power any other person might have. To the extent that everyone reaches a consensus on the reputa-

tional power of any individual, then the faith one can have in these reputational assessments increases.

To assess this assumption of consensus, or reliability, Cronbach's α was calculated for each of the two reputational measures, charisma and potency. Cronbach's α is an overall measure of agreement on a variable of interest, in this case, reputational power. The formula for Cronbach's α is provided by Carmines and Zeller (1979: 44):

$$\alpha = \frac{N\rho}{1 + \rho(N - 1)}$$

where N is the number of scores an individual received (or 33, one from each respondent who filled out the questionnaire), and ρ is the average interrater correlation, or agreement, between all distinct pairs of the 33 respondents. An individual correlation between two raters was determined by correlating their ratings of the 36 people in the firm on their reputational power. The score is high to the extent that everyone agrees (i.e., there is small variance) on each person's power score relative to the distribution of scores received by individual people.

Both charisma and potency had high reliability coefficients (Cronbach's α = .96 and .99, respectively), demonstrating that there was very high consensus in the organization on who was influential on each of these dimensions. The correlation between the two power indicators was .63, indicating considerable overlap between the two measures. For this reason, the two measures were combined into a single dependent variable, overall power, using the factor scores from the first component of a principal components analysis (PCA) of the two variables.[6]

The means, standard deviations and intercorrelations among all the variables used in this study are presented in Table 2.[7]

To test more completely the model in Figure 1, a set of hierarchical regressions was performed on the dependent variable, overall power. The results are presented in Table 3 as reduced form equations (Cohen and Cohen, 1983: 361–366).[8] Formal position explains 43 percent of the variance in overall power. The two informal structure sources of power, centrality in the advice and friendship networks, add another 17 percent of explained variance (significant at the .007 level). Note, however, that advice centrality is not significant in the equation; only centrality in the friendship network is significantly related to power when controlling for formal position ($p < .01$). It appears, then, that any advantage a person has by being central in the routine advice network is attributable to his or her formal position of power in the organization.[9]

In equation 3 of Table 3, cognitive accuracy on the advice and friendship networks explains an additional 8.2 percent variance ($p < .047$). Again, however, only one of the two added variables is significant: accuracy on the advice network. Understanding the friendship network is not significantly related to one's power reputation over and above being in the center of the networks and having a position of formal authority.

6

The factor scores of a simple, two-variable PCA are a linear function of the sum of the *z*-scores of each variable; that is, each variable contributes equally to the variance in the factor scores. Whenever one performs a PCA on two variables, the first eigenvalue will always = 1 + *r* (1.628, in the present case), and the variance explained by this factor will be 1 + *r*/2 (here, .814).

7

As a supplemental overall test of the proposition, a full canonical analysis was performed on all seven variables. Potency and charisma were the dependent variables; cognitive accuracy in both the advice and friendship network were the independent variables; and the controlling variables, centrality in each of the informal networks and formal position, were partialled out of the canonical analysis. While this analysis produced two canonical factors, only the first of them was significant ($p < .05$). The canonical correlation was equal to .455, over three times its standard error (.147).

8

Statistical tests in this paper are based on the usual OLS estimates of second moments. Such tests assume that observations are independent of one another. Recent literature questions this assumption in social science phenomena (Doreian, 1980; Dow, Burton, and White, 1982; Doreian, Teuter, and Wang, 1984). This problem has been found to be particularly severe in network research when the unit of analysis is the dyad (Krackhardt, 1988). Here, however, this network autocorrelation problem is attenuated by the fact that the dyadic data are aggregated to the individual-actor level of analysis before statistical tests are performed, and this Krackhardt-type problem is therefore not encountered.

9

At the recommendation of one reviewer, these analyses were replicated using closeness centrality in place of betweenness centrality. The argument for closeness centrality is that it may be a better predictor of cognitive accuracy, since it represents that average path distance an actor is away from everyone else on the average. Using closeness, the results paralleled those found in Table 3, except that closeness was a weaker predictor than betweenness had been. In fact, closeness in the advice network was negatively, albeit insignificantly, correlated with advice accuracy ($r = -.17$). Substituting closeness for betweenness in equation 2 of Table 3 results in an R^2 of .520 (vs. .597 when betweenness is used). While the strength of the predictions was reduced, each of the coefficients that were significant in Table 3 remained significant when closeness was used in place of betweenness.

Table 2

Means, Standard Deviations, and Correlations among Variables (N = 33)

Variable	Mean	S.D.
Power	0	1.00
Advice accuracy	.406	.0624
Friend accuracy	.326	.0704
Advice centrality	14.443	27.3039
Friend centrality	17.014	25.1941
Formal position	1.333	.6454

		Correlation structure			
Variable	Power	1	2	3	4
1. Advice accuracy	.340•				
2. Friend accuracy	.146	.282•			
3. Advice centrality	.453••	.210	.031		
4. Friend centrality	.506••	.172	.236	.222	
5. Formal position	.656•••	.240	.041	.566•••	.161

• $p < .05$; •• $p < .01$; ••• $p < .001$.

However, an understanding of the advice network is significantly related to one's power reputation.

These results reveal an interesting juxtaposition of effects. Clearly, formal authority is correlated with reputed power, as expected, but the two networks relate in different ways to one's power base. Centrality in the friendship network—not

Table 3

Hierarchical Regression Analysis of Reduced-Form Equations with Reputational Power as Dependent Variable*

	Equation		
Independent variables	(1)	(2)	(3)
Formal position	1.017•••	.879•••	.782•••
	(.210)	(.222)	(.210)
Advice centrality		.0015	−.0004
		(.005)	(.005)
Friend centrality		.161••	.0195•••
		(.0048)	(.0049)
Advice accuracy			5.091••
			(2.02)
Friend accuracy			−.559
			(1.74)
R^2	.431	.597	.678
Hierarchical test of model			
ΔR^2		.166	.082
F		5.966	3.425
d.f.		2,29	2,27
p		.007	.047

• $p < .05$; •• $p < .01$; ••• $p < .001$.
* Standard errors are in parentheses.

the advice network—is a key factor in reputed power; but it is cognitive accuracy of the advice network—and not the friendship network—that adds a significant amount of explained variance to one's power reputation.

A closer inspection of the simple relationships among the variables in Table 2 provides a partial explanation for these findings. Advice centrality and friendship centrality are both strong simple predictors of power (.45 and .51, respectively). But, while advice centrality is correlated with formal authority (.57), friendship centrality is not significantly related (.16). Most of the variance in power explained by advice centrality is already explained by formal authority: those central in the advice network are also those with higher authority. Since friendship centrality is not related to formal authority, however, it provides a unique contribution to power in the second step of the hierarchical regression. Knowledge of the advice network does not significantly covary with formal authority and therefore also provides a unique contribution to power in the third step of the regression.

DISCUSSION

The network analysis conducted on Silicon Systems confirms the major proposition of this study, that an accurate picture of the informal network significantly correlates with power. But, the overall model presented in Figure 1, relating structural factors to cognition and power, received only qualified support (see Table 2). As predicted, formal position is significantly related to power and advice centrality. Contrary to the model's predictions, formal position does not significantly correlate with cognitive accuracy. Also, contrary to prior research (Freeman and Romney, 1987), centrality was not directly related to cognitive accuracy. Since these simple correlations were not confirmed in Table 2, more elaborate tests of the path coefficients for Figure 1 were not necessary, beyond the overall tests provided for by the hierarchical regressions reported in Table 3.

The question remains why the other relationships in Figure 1, which form the theoretical building blocks for the basic proposition of this paper, were not confirmed. One possible explanation for the lack of support for parts of the causal model may rest in the size of the firm. Because the firm is small, people all know each other and are relatively better informed on each other's relationships than they might be in a large organization. Thus, being in the center of the network or at the top of the formal hierarchy does not provide as strong an informational edge over others' vantage points. Perhaps in a larger organization, where many people are not even aware of each other's existence, these structural advantages may prove more predictive of cognitive accuracy. Future research on this topic could shed light on whether these results are generalizable to—or perhaps even enhanced in—larger firms.

Since the structural links to cognitive accuracy were not confirmed, the test of the major proposition of this study could have been reduced to a simple correlation between cognitive accuracy and reputational power without controlling for formal or informal positional power. It was necessary, nonetheless, to present the analysis in full, controlling for these theoreti-

cally important sources of power. The amount of variance in power explained by cognitive accuracy in the advice network alone was 11.6 percent ($=.341^2$); the direct contribution of the two accuracy indicators in equation 3 in the hierarchical analysis was 8.2 percent. This difference indicates that, as predicted by the theory, there is some, albeit small, spurious correlation due to structural effects. In the conservative approach taken here, the hierarchical test of the main proposition removes this spuriousness.

The study showed that reputational power of the members of the firm was significantly related to cognitive accuracy of the advice network, not the friendship network. Perhaps this is an indication of the extent to which power surrounded those who were capable of handling relatively routine operational problems. In answering questions about influence and power, employees were responding according to their experiences in their day-to-day lives in the organization. As mentioned above, those people in the "advice" network, the experts, are likely to derive power from such routine situations. Had the organization faced a nonroutine situation such as a crisis, however, it is possible that an understanding of the friendship network could have been more predictive of power in dealing with the crisis. Dealing with crises does not require routine information but, rather, it requires trust (Krackhardt and Stern, 1988). It is reasonable to speculate that understanding the friendship network, which better represents the trust relations in the organization, could prove more critical than understanding the advice network in such a nonroutine situation. Of course, this is only speculation, since the data reported here do not involve anything but routine operations.

Caveats and Limitations

The theory presented at the beginning of the paper argued that knowledge of the network is in its own right a base of power above and beyond the power accrued through other formal and informal bases. This causal claim leads directly to the prediction of association. However, as is always the case in field studies such as this, one cannot infer the causal link from the data. There are three possible reasons for an association between two variables, A and B: A leads to B, B leads to A, or there is a third variable (or set of variables) that leads to both A and B, in which case we say that the relationship is spurious. It is worth speculating about each of these possible reasons for the underlying association.

In the current study, the theoretical claim underlying the observed association is that network knowledge leads to reputational power. But is it possible that one's reputation as a powerful person leads to a better understanding of the social network? Perhaps, for instance, as one becomes reputed to have more power, one is fed differentially more social information. However, if this were the case, then it is likely that this differential focus of information would in turn lead to the actor becoming central in the network, and partialling out network centrality would remove the association between reputational power and network knowledge.

A more serious concern is whether the observed relationship is spurious. Despite some statistical attempts to control for clear sources of spuriousness, there are potentially an infinite

number of variables that are unaccounted for. For example, suppose that power reputation is an attribution based on the fact that certain people are closer to the action in the organization. Suppose that being closer to the action also gives people certain advantages in knowing the social network. Then one could argue that the observed relationship between reputational power and network knowledge is spurious. In part, one could also argue that being "closer to the action" is already controlled for by controlling for centrality in the network; but then again, it may not control for all of it. I have controlled for what I argued to be the most reasonable sources of spuriousness. But, clearly, one cannot conclude that all sources of spuriousness have been eliminated. One reason for including the raw data in this paper was to invite scholars to explore alternative models that might explain the reported relationships.

Power. Another important issue surrounds the use of the term "power" in this study. The literature on power in organizations is extensive, and on a theoretical level, this study sheds light on only a small part of that literature. The emphasis here is on the power induced through information flows in the informal network. Dependencies and power in an organization can emanate from many sources, not simply how information is passed from one person to the next. For example, I have no indication of the important workflow interdependencies (cf. Brass, 1984). Nor am I able to locate the critical resources or who has control over them (cf. Pfeffer and Salancik, 1978). Nor do I have information on who is performing the important functions in the organization and the exclusivity with which they perform them (see Dubin, 1957: 62, for the first explicit treatment of this two-component definition of power). My use of the term "power" in this study is relatively specialized and may not generalize to other conceptualizations or other contexts. Whether understanding the social network has any bearing on whether an individual has control over critical resources is an interesting question that must be left to future research.

Secondly, there is a methodological limitation to this study, one discussed in full by Pfeffer (1981: 54–57). Using reputation to measure the relative power of an individual has potential biases. The measure assumes that the raters know who is powerful and that they are willing to tell the researcher honestly what they know. Despite these possible problems, Pfeffer (1981: 56–57) noted that when raters seem to agree on their power attributions, "this consensus and consistency in power ratings provides some evidence for at least a shared social definition of the distribution of power." Given the high reliability scores of the components of the reputational measure in this study, I share Pfeffer's conclusion that at least I have demonstrated that there is consensus in the power attributions.

CONCLUSION

This paper demonstrates that knowledge of the relevant network is itself associated with reputational power, independent of other structural bases of power. In particular, further work exploring the importance of the structure of different

kinds of relations in organizations may prove fruitful in understanding the dynamics of organizational behavior.

As Mintzberg (1983: 1) put it, "Power is a major factor, one that cannot be ignored by anyone interested in understanding how organizations work and end up doing what they do." The contribution in this paper is to focus on how political actors themselves gain power by following in the footsteps of organizational scientists in understanding and identifying the power bases in the political arenas in which they operate. A meta-power emerges here: the power of power. If power is the ability to influence a target, then meta-power is an indirect power derived from knowing and using the power others have to influence the target. Given all the attention paid to structural and resource bases of power, it is surprising that so few have investigated the power due to such political knowledge.

REFERENCES

Bachrach, P., and M. S. Baratz
1962 "The two faces of power." American Political Science Review, 56: 947–952.

Bailey, F. G.
1969 Strategems and Spoils: A Social Anthropology of Politics. Oxford: Blackwell.

Bavelas, Alexander
1948 "A mathematical model for group structure." Human Organization, 7: 16–30.

Bernard, H. Russell, Peter Killworth, David Kronenfeld, and Lee Sailer
1984 "The problem of informant accuracy: The validity of retrospective data." Annual Review of Anthropology, 13: 495–517.

Bierstadt, Robert
1950 "An analysis of social power." American Sociological Review, 15: 730–738.

Bonacich, Philip
1987 "Power and centrality: A family of measures." American Journal of Sociology, 92: 1170–1182.

Bradley, Raymond T.
1987 Charisma and Social Structure: A Study of Love and Power, Wholeness and Transformation. New York: Paragon.

Brass, Daniel J.
1984 "Being in the right place: A structural analysis of individual influence in an organization." Administrative Science Quarterly, 29: 518–539.

Burt, Ronald S.
1982 Toward a Structural Theory of Action. New York: Academic Press.

Carmines, Edward G., and Richard A. Zeller
1979 Reliability and Validity Assessment. Beverly Hills, CA: Sage.

Cohen, Jacob, and Patricia Cohen
1983 Applied Multiple Regression/Correlation Analysis for the Behavioral Sciences. Hillsdale, NJ: Erlbaum.

Doreian, Patrick
1980 "Linear models with spatially distributed data: Spatial disturbances or spatial effects?" Sociological Methods and Research, 9: 29–60.

Doreian, Patrick, Klaus Teuter, and Chi-Hsein Wang
1984 "Network autocorrelation models: Some Monte Carlo results." Sociological Methods and Research, 13: 155–200.

Dow, Malcolm M., Michael L. Burton, and Douglas R. White
1982 "Network autocorrelation: A simulation study of a foundational problem in regression and survey research." Social Networks, 4: 169–200.

Dubin, Robert
1957 "Power and union management relations." Administrative Science Quarterly, 2: 60–81.

Emerson, Richard M.
1962 "Power-dependence relations." American Sociological Review, 27: 31–41.

Fiedler, Fred, and Robert J. House
1988 "Leadership theory and research: A report of progress." In Cary L. Cooper and Ivan T. Robertson (eds.), International Review of Industrial and Organizational Psychology: 73–92. New York: Wiley.

Freeman, Linton C.
1979 "Centrality in social networks: Conceptual clarification." Social Networks, 1: 215–239.

Freeman, Linton C., Sue C. Freeman, and Alaina G. Michaelson
1988 "On human social intelligence." Journal of Social and Biological Structures, 11: 415–425.

Freeman, Linton C., and A. Kimball Romney
1987 "Words, deeds and social structure." Human Organization, 46: 330–334.

Freeman, Linton C., A. Kimball Romney, and Sue C. Freeman
1987 "Cognitive structure and informant accuracy." American Anthropologist, 89: 310–325.

French, John R. P., and Bertram Raven
1968 "The bases of social power." In Dorwin Cartwright and Alvin Zander (eds.), Group Dynamics, 3d ed.: 259–269. New York: Harper & Row.

Gower, J. C., and Pierre Legendre
1986 "Metric and Euclidean properties of dissimilarity coefficients." Journal of Classification, 3: 5–48.

House, Robert J.
1977 "A 1976 theory of charismatic leadership." In J. G. Hunt and L. L. Larson (eds.), Leadership: The Cutting Edge: 189–207. Carbondale, IL: Southern Illinois University Press.

Hubbell, Charles H.
1965 "An input-output approach to clique identification." Sociometry, 28: 377–399.

Kanter, Rosabeth Moss
1979 "Power failure in management circuits." Harvard Business Review, July/August: 65–75.

Krackhardt, David
1987 "Cognitive social structures." Social Networks, 9: 109–134.
1988 "Predicting with networks: Nonparametric multiple regression analysis of dyadic data." Social Networks, 10: 359–381.

Krackhardt, David, and Lyman W. Porter
1985 "When friends leave: A structural analysis of the relationship between turnover and stayers' attitudes." Administrative Science Quarterly, 30: 242–261.
1986 "The snowball effect: Turnover embedded in communication networks." Journal of Applied Psychology, 71: 50–55.

Krackhardt, David, and Robert N. Stern
1988 "Informal networks and organizational crises: An experimental simulation." Social Psychology Quarterly, 51: 123–140.

Kruskal, Joseph B., and Myron Wish
1978 Multidimensional Scaling. Beverly Hills, CA: Sage.

Laumann, Edward O., and Franz U. Pappi
1976 Networks of Collective Action: A Perspective on Community Influence Systems. New York: Academic Press.

Lincoln, James R., and Jon Miller
1979 "Work and friendship ties in organizations: A comparative analysis of relational networks." Administrative Science Quarterly, 25: 181–199.

Lukes, S.
1974 Power: A Radical View. London: MacMillan.

McCelland, David C.
1975 Power: The Inner Experience. New York: Irvington.

Mintzberg, Henry
1983 Power in and around Organizations. Englewood Cliffs, NJ: Prentice-Hall.

Mizruchi, Mark S.
1983 "Who controls whom? An examination of the relation between management and boards of directors in large American corporations." Academy of Management Review, 8: 426–435.

Pettigrew, Andrew M.
1973 The Politics of Organizational Decision-Making. London: Tavistock.

Pfeffer, Jeffrey
1981 Power in Organizations. Boston: Pitman.

Pfeffer, Jeffrey, and Gerald Salancik
1978 The External Control of Organizations. New York: Harper & Row.

Roberts, Nancy
1986 "Organizational power styles: Collective and competitive power under varying organizational conditions." Journal of Applied Behavioral Science, 22: 443–458.

Salancik, Gerald R.
1986 "An index of subgroup influence in dependency networks." Administrative Science Quarterly, 31: 194–211.

Salancik, Gerald R., and Jeffrey Pfeffer
1977 "Who gets power and how they hold onto it: A strategic-contingency model of power." Organizational Dynamics, 5: 3–21.

Stephenson, Karen, and Marvin Zelen
1989 "Rethinking centrality: Methods and examples." Social Networks, 11: 1–37.

APPENDIX: Cognitive Social Structure of Advice Network

Below are the raw cognitive social structure (CSS) scores for the advice network for all 33 respondents who completed the questionnaire. The number to the left of the equal sign is the ID number of the respondent. The 36 × 36 matrix represents that respondent's slice of the CSS. The diagonals of each slice are undefined and are so indicated with an *x*.

$$1 = \begin{pmatrix}
x00101000000000001100001001000000000\\
0x00100000000000000000000000010000000\\
00x01100000000000010000000010000000000\\
000x00000000000001101000001000000000\\
0000x00000000000010000000000000000000\\
00001x00000000000010000000010000000000\\
000101x00000000001100000001010000000\\
0000100x00000000000000000000000000000\\
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0000100000000000000010000000010000x00\\
000000000000000010010000000000000000x1\\
000000000000000010010000000000000000x
\end{pmatrix}$$

$$2 = \begin{pmatrix}
x00100000000000001000001001000000000\\
0x01100000000000000000000000010000000\\
00x01100000000010010000000000000000\\
100x00000000000001100000001000000000\\
0000x10000000001001000000001000000000\\
00100x00000000010010000000000000000\\
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0000110000000001001000000x0000000010\\
1001100000000001011000010 0x000000000\\
000001000000100100100000100x00100001\\
0000100000000000001000000000x0000000\\
0000100010000000101000000000x000000\\
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0100101100100110000100100000100x1000\\
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0000110000000001001000000000000000x0\\
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\end{pmatrix}$$

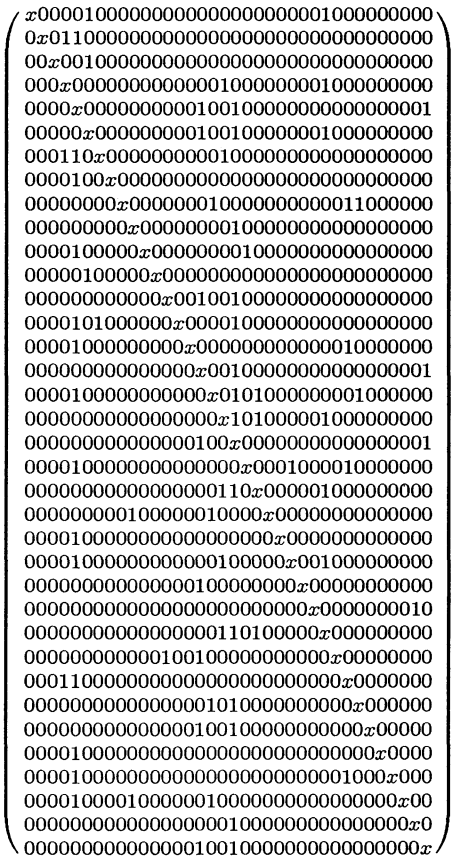
$$3 = \begin{pmatrix} x0000100000000000010000001000000000 \\ 0x0010000000000000010000000010000000 \\ 00x0110000000000000000000001000000000 \\ 000x0000000000000110000000010000000000 \\ 0000x1000000000000101000000000000000000 \\ 00001x0000000000001000000000000000000 \\ 000100x0000000000011000000000000000000 \\ 0000100x0000000000010000000000000000 \\ 00000000x1000000100000000000000100000 \\ 000000001x0000001000000000000000100000 \\ 0000100000x0000000010000000010000000 \\ 00100100000x0000000000000010000000000 \\ 000010000000x0010010000000000000000000 \\ 000010110000000x0000010100000010001000 \\ 000010000000000x0000010100000010000100 \\ 000001000000000x0010000000000000100001 \\ 000001000000000x0101000000001000000 \\ 000000000000000000x1000000010000000000 \\ 000000000000000000x0000000000000000000 \\ 000011000000000001x0000000010001000 \\ 000000000000000001010x000000000000000 \\ 000000000100000010100x00000001000000 \\ 010000110010011000010x0000010010000 \\ 000110000000000011000x001001000000 \\ 000011000000100100100000x00000100000 \\ 000001000000000000000000x0000000010 \\ 000000000000000000100000x000000000 \\ 000001000000100100000000100x00000000 \\ 000010000000000001000000000x0000000 \\ 000000000000000001010100000000x000000 \\ 000000000000000100100000000000x00001 \\ 010000010000011000010010000010 0x0000 \\ 000000000000000000010100000010001000x100 \\ 000000000000000000000100000011000x00 \\ 000001000000000100100000000000000x1 \\ 000000000000000100100000000000000x \end{pmatrix}$$

$$4 = \begin{pmatrix} x0000100000000000000000000000000000 \\ 0x0010000000000000000000000000000000 \\ 00x0010000000000000000000000000000000 \\ 000x0000000000000000000000100000000000 \\ 0000x0000000000000100000000000000000 \\ 00000x0000000000001000000000000000000 \\ 000010x0000000000000000000000000000000 \\ 0000100x0000000000000000000000000000 \\ 00000000x0000000000000000000100000000 \\ 000000000x0000001000000000000000000000 \\ 0000100000x0000000000000000000000000 \\ 0000010000000x0010000000000000000000000 \\ 000000000000x0010000000000000000000000 \\ 0000100000000x0000000000000000000000000 \\ 0000010000000x0000000000000000000000000 \\ 000000000000000x0100000000000000000000 \\ 000000000000000x0000000010000000000 \\ 000010000000000x0000000000000000000 \\ 000010000000000000x0000000000000000000 \\ 000000000000000000x0000010000000000 \\ 000000000000000010000x0000000000000 \\ 000010000000000000000x000000000000 \\ 000010000000000000000x0000000000000 \\ 000000000000100000000000x00000000000 \\ 000000000000000100000000000x0000000000 \\ 000001000000000000000000000x000000000 \\ 000000000000000010000000x0000000000 \\ 000000000000100000000000000x00000000 \\ 000010000000000000000000000x0000000 \\ 000000000100000000000000000x000000 \\ 000000001000000000000000000000x00 \\ 000000000000000010000000000000000x0 \\ 000000000000000100100000000000000000x \end{pmatrix}$$

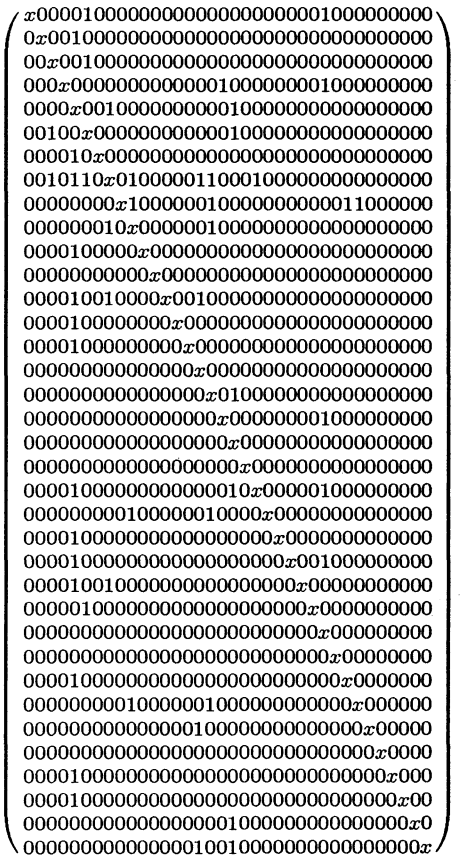
$$5 = \begin{pmatrix} x0000100000000000000000000000000000 \\ 0x0010000000000000010000000000000000 \\ 00x0010000000000000000000000000000000 \\ 000x0000000000000000000001000000000 \\ 0000x0000000000001000000000000000000 \\ 00001x0000000000001000000000000000001 \\ 000010x0000000000000000000000000000000 \\ 0000100x0000000000000000000000000000 \\ 00000000x0000001000000000001000000 \\ 000000000x0000010000000000000000000 \\ 0000100000x0000000000000000010000000 \\ 0000010000000x0010000000000000000000 \\ 0000101000000x0000000000000000000000 \\ 0000100000000000x0000000000010000000 \\ 0000100000000001x0100000000000000001 \\ 0000100000000001x0100000000000000001 \\ 000000000000000000x1000000010000000000 \\ 0000100000000000110x0000000000000000001 \\ 0000100000000000000x0000000010000000 \\ 0000100000000000000010x0000000000000000 \\ 0000100001000000000000x00000000000000 \\ 0000100000000000000000000x0000010000000 \\ 0000100000000000000010000x001000000000 \\ 0000100000010010010000000x00000000000 \\ 0000010000000000000000000000x0000000000 \\ 0000000000000000000100000000x000000000 \\ 0000000000001001000000000100x00000000 \\ 0000100000000000000000000000000x0000000 \\ 0000000000000000101000000000000x000000 \\ 0000000000000001001000000000000000x00000 \\ 0000100000000000000000000000000100x1000 \\ 0000100000000000000000000000001000x000 \\ 0000100001000000000000000000000000000x00 \\ 0000000000000000100100000000000000x1 \\ 0000000000000000100100000000000000000x \end{pmatrix}$$

$$6 = \begin{pmatrix} x0000100000000000110000000100000010 \\ 0x0010000100000000000000000000000000 \\ 00x0010000000000001000000000000000000 \\ 000x1000000000001100000001000000000 \\ 0000x0000000000001000000000000000000 \\ 00001x0000000000001010000000000000000 \\ 000010x0000000000000000000000000000000 \\ 0010110x0000000000000000000000000000 \\ 00000000x0000000000010000000110000000 \\ 010000000x0000001000000000000000000000 \\ 0000100000x0000000000000000000000000 \\ 0010010000x0000000000000000000000010 \\ 0000100000000x0010110000000100000000 \\ 0000100000000x0000000000000000000000 \\ 0000100000000x0000000000000000000000 \\ 000000000000000x0010000000000000001 \\ 0000100000000000x0100000000000000000 \\ 0001000000000000x1000000000000000000 \\ 0000100000000000100x0000000000000001 \\ 0000100000000000000x0000000010000000 \\ 0000100000000000000010x0000000000000000 \\ 0000100000000000000000x00000000000000 \\ 0000100000000000000000000x0000000000000 \\ 0000110000000000000010000x001000000000 \\ 0000010000001001001000000x00000100000 \\ 0000010000010000000010000x0000000010 \\ 1000010000000000011000000x000000000 \\ 0000000000001001000000000100x00000000 \\ 0000100010000000001000000000x0000000 \\ 0000000000000000001000000000x000010 \\ 0000010000000001001000000000x00000 \\ 0000100000000000000000000000000x0000 \\ 0000100000000000000000000000000000x000 \\ 0000100000000000000000000000000000000x00 \\ 0000010000000001011000000000000000x0 \\ 0000000000000001001000000000000000000x \end{pmatrix}$$

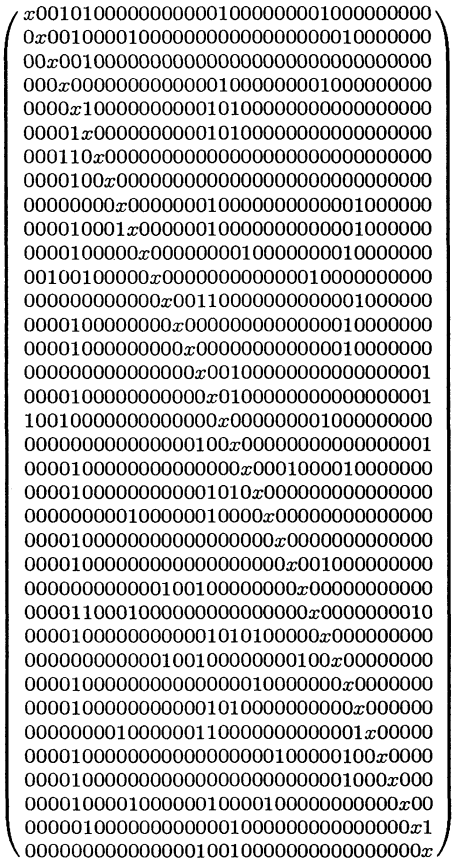
$7 =$



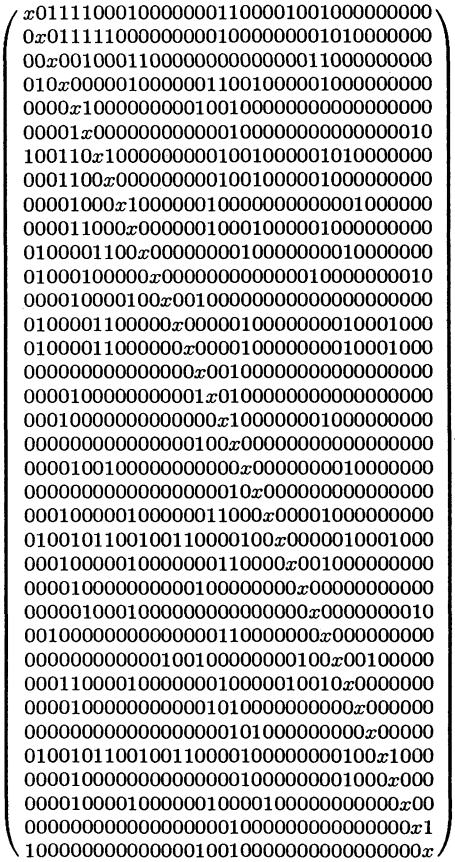
$8 =$



$9 =$



$10 =$



$$11 = \begin{pmatrix}
x0000100000000000000000000001000000010\\
0x0010000000000000010000000010000000\\
00x0010000000000000000000000000000000\\
000x0000000000000100000000100000000\\
0000x0000000000001000000000000000000\\
00000x0000000001001000000000000000000\\
000010x0000000000001000000101000000\\
0000100x0000000000001000001000000000\\
00001000x1000000100000000000001000000\\
000000001x0000001000000000001000000\\
0000000000x0000000000000000000000000\\
00100100000x0000000000000010000000010\\
0000100000000x0010010000000000000000\\
0000101000100x0000010000000010000000\\
0000100000000x0000010000000010000000\\
0000000000000000x0010000000000000000\\
0000100000000000x0101000000000000000\\
0001100000000000x1000000010000000000\\
0000110000000000100x0000000000000000\\
0000100000000000000x0000001011000000\\
0000100000000000010x0000000000000000\\
0000100001000000010000x00000000000000\\
0000101000000000000100x0000010000000\\
0000100000000000000000x0010000000000\\
0000000000001001000000000x00000000000\\
0010010000000000000000000x0000000010\\
0001100000000000011000000x000000000\\
0000000000001001000000000000x00000000\\
0000100000000000000100000000x0000000\\
0000100011000000100010000100x000000\\
0000010000000001000000000000000x00000\\
0000100000100001000000000000100x1000\\
0000001000100000000100000000001000x000\\
0000100001000000010000100000001000x00\\
0000010000000000001000000000000000x0\\
0000000000000000100100000000000000000x
\end{pmatrix}
\qquad
12 = \begin{pmatrix}
x0111111000100000110000100100000010\\
0x1011110000011000110110010010011110\\
11x1111100111110011101101111100111110\\
101x0100110100001110100101100100010\\
0110x1110011011000110110011010011110\\
10101x0000001000101000010111000000010\\
111111x1001101100111001101101011110\\
1011110x0001000001101000011000000010\\
10010100x1000000111000000010010000010\\
100101001x0000001110000101100100010\\
0110111100x0011000110110010010011110\\
10111100000x1000011000011111000000010\\
101001000001x0010010000011010010011\\
0110111100100x1000110110010010011110\\
0110111100100x1000110110010010011110\\
000001000000100x0010000000100100011\\
100001001100000x0100001001000000010\\
101101001101000001x1000010110010000010\\
100011001100000110x0000101100110011\\
011011100010011000 1x0111010010011110\\
000011001100000010 10x0000100010000010\\
011011100010011000110x1001001001111 0\\
011011100010011000 1101x0010110011110\\
101111001101000011 10100x0110010000 10\\
000001000001100100 100000x10100100011\\
111011100011111100 1101101x0010111111\\
101101001101000011 10100101x0010000 10\\
000001000001100100 1000000 10x00100011\\
011011100010011000 1101100100x0011110\\
100101001100000011 1000010010 0x000010\\
000001000000100100 1000001101 00x00011\\
011011100010011000 1101100100100x1110\\
011011100010011000 1101100100100 1x110\\
011011100110011000 1101100100100 11x10\\
000001000000000100 1000000100000000x1\\
000001000000100100 1000000000010001x
\end{pmatrix}$$

$$14 = \begin{pmatrix}
x0000000000000000100000001000000000\\
0x0010000000000000000000000000000000\\
00x0000000000000000000000000000000000\\
000x0000000000000000000000000000000\\
0000x0000000000001000000000000000000\\
00000x0000000000010000000000000000000\\
000010x0000000000000000000000000000\\
0000100x0000000000000000000000000000\\
00000000x1000000000000000000000000000\\
000010000x0000000000000000000000000\\
0000100000x0000000000000000000000000\\
00000000000x0000000000000000000000000\\
000000000000x0010000000000000000000\\
0000100000000x0000000000000000000000\\
0000100000000x0000000000000000000000\\
0000000000000000x0010000000000000000\\
0000000000000000x0000000000000000000\\
0000000000000000x1000000000000000000\\
0000000000000000100x000000000000000000\\
0000100000000000000x00000000000000000\\
0000000000000000000x0000000000000000\\
0000100000000000000x0000000000000000\\
0000100000000000000x0000000000000000\\
0000000000000000000000x000000000000\\
0000000000000000000000000x00000000000\\
0000010000000000000000000x0000000000\\
0000000000000000000000000x000000000\\
0000000000000000000000000000x00000000\\
0000100000000000000000000000x0000000\\
0000100000000000000000000000000x000000\\
0000010000000000000000000000000000x00\\
0000000000000000010000000000000000x0\\
0000000000000000100100000000000000000x
\end{pmatrix}
\qquad
15 = \begin{pmatrix}
x0000100000000000000000000001000000000\\
0x0010000000000000000000000010000000\\
00x0110000000000000000000000000000000\\
000x0000000000001001000001000000000\\
0000x0000000000010101000001000000000\\
00101x0000000000001000000001000000000\\
001110x0000000100100000000101000000\\
0000000x0000000000000000000000000000\\
00000000x1000000100000000001000000\\
100010001x0000010000100001011000000\\
0000100000x0001000010001000010000000\\
10000100000x0000000000000000000000010\\
0000011100000x0110101100000010101000\\
0000101000000x1000010000000010000000\\
0000101000000x0000010010000010000000\\
0000100000000100x0010000000100010001\\
0000100000000000x0101000001001000000\\
0001100000000000x1010000001000000000\\
0000100000000000100x0000000001000000001\\
0000100000000000000x0001001010000000\\
0000100000000000000110x000001000000000\\
0010110000000000010001x01001010000000\\
0100101000000010000110x1000010000000\\
1011110000000000100100x001000000010\\
0000110000001001001000000x01100100000\\
1000010000000000000000000x0000000010\\
0010110000000000001010000000x000000000\\
0000010000001001000000000000x00100000\\
0110110000000010011010000000x0000000\\
0000100000000000101000000000000x000000\\
0000000000000000100100000000000x00001\\
0100101000000010001100000000100x0000\\
0010101000000010000000000001000x000\\
0010100001000000000001000000000x00\\
0000010000000000001000000100000000x0\\
0000000000000001001000000000000000x
\end{pmatrix}$$

$$16 = \begin{pmatrix}
x00101000000000011100001001100100000 \\
0x0010000000000000000000000000000000 \\
00x001000000000000000000000000000000 \\
100x01000000000011100001001000000010 \\
0000x1000000000110101000000000000001 \\
00001x000000000110100000000000000011 \\
000010x00000000000000000000000000000 \\
0000100x0000000000000000000000000000 \\
00000100x000000010100000000001000000 \\
000000000x00000010100000000001000000 \\
0000100000x0000000000000000000000000 \\
00000100000x000000000000000000000000 \\
000011000000x00110100000101000100001 \\
0000100000000x0000000000000000000000 \\
00001000000000x000000000000000000000 \\
000011000000100x10100000101101100011 \\
0000100001000000x0100000000001000001 \\
10010000000000000x100001001000000000 \\
000011000000000100x01000001000000011 \\
0000100000000000000x0000000000000000 \\
00001000000000011010x000000000000000 \\
000010000000000000000x00000000000000 \\
0000100000000000000000x0000000000000 \\
10011000000000001110000x001000000000 \\
000011000000100110100000x00000100001 \\
0000010000000000000000000x0000000000 \\
10000000000000001010100000x001000000 \\
000000000000100100100000100x00000001 \\
0000100000000000000100000000x0000000 \\
00001000000000011010000000000x000001 \\
000011000100100110101000100000x00001 \\
0000100000000000000000000000000x0000 \\
00001000000000000000000000000000x000 \\
000010000000000000000000000000000x00 \\
0000000000000001001000000000000000x1 \\
00000100000000011010000000000000000x
\end{pmatrix}$$

$$17 = \begin{pmatrix}
x00101000000000000100000000100000000 \\
0x0000000000000000001000000010000000 \\
10x001000000000000000000000000000000 \\
000x00000000000000110000000100000000 \\
0000x0000000000001010000000001000000 \\
00001x000000000000100000000000000000 \\
000010x00000000000000000000000000000 \\
0000100x0000000000000000000000000000 \\
00000000x100000000000000000001000000 \\
000000000x00000010000000000001000000 \\
0000100000x0000000001000000010000000 \\
00000100000x000000000000001000000000 \\
000010000100x00110000000000000000000 \\
0000100000000x0000100000000010000000 \\
00001000000000x000100000000010000000 \\
000010000000000x10100000000001000000 \\
0000100000000000x0101000000100000000 \\
00000000000000000x100000001000000000 \\
000010000000000110x01000000010000001 \\
0000100000000000000x0000000010000000 \\
00001000000000001010x000000001000000 \\
000000000100000010000x00000000000000 \\
0000100000000000000100x0000010000000 \\
00001000000000001010000x001001000000 \\
000011000100000110000000x00000000000 \\
0000010000000000000000000x0000000000 \\
00000000000000001000000000x000000000 \\
000010000001001000000000000x00000000 \\
0000100000000000001000000000x0000000 \\
00001000000000001010000000000x000000 \\
000000000000000110100000000000x00000 \\
0000100000000000110000000000000x0000 \\
00001000000000000010000000001000x000 \\
000000000100000010000100000000000x00 \\
0000000000000000001000000000000000x1 \\
00000000000000011010000000000000000x
\end{pmatrix}$$

$$18 = \begin{pmatrix}
x00001000000000000000000001000000000 \\
0x0110000000000000100000001010000000 \\
00x001000000000000000000000000000000 \\
000x00000000000001001001001000000000 \\
0000x0000000000001010000000000000000 \\
00000x000000000000100000000000000000 \\
000110x00000000010000000001000000000 \\
0000100x0000000000000000000000000000 \\
00000000x100000010000000000001000000 \\
000000001x00000010000000000001000000 \\
0000100000x0000000000000000010000000 \\
10000100000x000000000000010000000000 \\
000000000000x00100000000000000000000 \\
0000101000000x0000000000000010000000 \\
00001000000000x000000000000010000000 \\
000000000000100x00100000100100100001 \\
0000000001000000x0001000000001000000 \\
00010000000000000x101000001000000000 \\
000010000000000100x00000000000000001 \\
0000100000000000000x0000000010000000 \\
00000000000000000100x000001000000000 \\
000010000100000000000x00000000000000 \\
0000100000000000000000x0000010000000 \\
00001000000000000000100x001000000000 \\
000000000000100100000000x00000100000 \\
0000010000000000000000000x0000000000 \\
00010000000000000100100100x000000000 \\
000000000000100100000000100x00000000 \\
0000100000000000000000000000x0000000 \\
00000000010000001000000000000x000000 \\
000000000000000000100000000000x00000 \\
0000100000000000000000000000100x0000 \\
00001000000000000000000000001000x000 \\
000010000100000000000000000000000x00 \\
0000000000000000001000000000000000x1 \\
000000000000000100100000000000000001x
\end{pmatrix}$$

$$19 = \begin{pmatrix}
x00001000000000001100000000100000000 \\
0x0010000000000010110000000010000000 \\
00x011000000000000100000000000000000 \\
000x10000000000011101001001001000000 \\
0000x1000000000110101000001001000000 \\
00001x000000000100100000000100000010 \\
000110x00000000000100000000100100000 \\
0000100x0000000000100000000010000000 \\
00000000x100000010100000000001000000 \\
000011000x00000011101000001001000000 \\
0000101000x0000000000000000010001000 \\
00000100000x000000000000000010000010 \\
000011000000x00111101000001011000000 \\
0000101000000x0000000000000010001000 \\
00001000000000x000000000001010000000 \\
000011000000100x01101000001010000001 \\
0000110001000001x1101000001001000001 \\
00000100000000001x100001001001000000 \\
000011000100000111x01001000001000001 \\
0000111001000000111x1000001011000000 \\
00001100000000001111x000001001000000 \\
000011000000000011101x01001011000000 \\
0000100000000000101100x0000010000000 \\
00011110000000001110100x001011000000 \\
000011000000000011010000x01001000001 \\
0000110000000000001000000x0000000010 \\
00011110000000001110100100x001000000 \\
000011000000100100100000110x00000000 \\
0001111000000000111110010010x1000000 \\
00001110000000001110100100100x000010 \\
000011000000000100100000000000x00000 \\
0000100000000000001000000000100x1000 \\
00001000000000001001000100101100x000 \\
000010000100000010000100000011001x00 \\
0000010000000001001000000010000000x1 \\
000011000000000100100000000000000001x
\end{pmatrix}$$

$$20 = \left(\begin{array}{c}
x00000000000000001001000001000000000\\
0x0010000000000000010000000010000000\\
00x001000000000000000000000000000000\\
000x00000000000001000000001000000000\\
0000x0000000000100110000000010000000\\
00000x000000000010010000000000000000\\
000010x00000000000111000001000000000\\
0000100x0000000000000000000000000000\\
00000000x100000010010000000010000000\\
000000001x00000010000000000001000000\\
0000100000x0000000010000000010000000\\
00000100000x000000000000001000000000\\
000000000000x00100110000000010000001\\
0000100000000x0000010000000010000000\\
00001000000000x000010000000010000000\\
000000000000000x00100000000000000000\\
0000000010000001x0100000000001000000\\
00010000000000000x000001001000000000\\
000010000000000100x10000000010000000\\
0000100000000001001x0000000010000000\\
00001000000000000011x000000010000000\\
000000001000000010010x00000011000000\\
0000100000000000000100x0000010000000\\
10010000000000000010000x001000000000\\
000010000000100100010000x00010000000\\
0000010000000000000000000x0000000010\\
10010000000000000100100000x000000000\\
000010000000100100010000100x10000000\\
0000100000000001001100000000x0000000\\
00000000110000001001000000001x000000\\
000000000000000100100000000000x00000\\
0000100000000000000100000000100x0000\\
00001000000000000001000000001000x000\\
000010001000000010010000000011000x00\\
0000000000000000000100000000000000x0\\
00001000000000011011000000101000000x
\end{array}\right)
\qquad
21 = \left(\begin{array}{c}
x00011000000000000101000001000000000\\
0x0010000000000000010000000010000000\\
00x011000000000000101000001000000000\\
000x11000000000001101000001000000000\\
0000x1000000000000101000000000000000\\
00000x000000000000101000000000000000\\
000010x00000000000010000000010000000\\
0000100x0000000000101000000000000000\\
00000000x000000010001000000001000000\\
000001000x00000010100000000001000000\\
0000100000x0000000000000000010000000\\
00000100000x000000000000001000000000\\
000011000000x00100100000000000000000\\
0000100000000x0000000000000010000000\\
00001000000000x000010000000010000000\\
000001000000000x00000000000000000001\\
0000000000000000x0100000000000000000\\
00000100000000000x101000001000000000\\
000011000000000000x01000000000000000\\
0000110000000000001x1000000000000000\\
00001100000000000010x000001000000000\\
000010000000000010111x00000011000000\\
0000100000000000000000x0000000010000\\
00011100000000000111100x001000000000\\
000011000000000100001000x01000000000\\
0000010000000000000000000x0000000010\\
00000000000000000010100000x000000000\\
000000000000100100000000100x00000000\\
0000110000000000001010000000x1000000\\
00001000000000001010100000000x000000\\
000001000000000100100000000000x00000\\
0000000000000000000000100000100x0000\\
00001000000000000001000000001000x000\\
000011000000000010100100000010000x00\\
0000010000000000000100000010000000x0\\
00000000000000010000000000000000000x
\end{array}\right)$$

$$22 = \left(\begin{array}{c}
x00101000000000001100001001000000000\\
0x0110100000000000100000001010000000\\
00x011000000000000000000000000000000\\
000x10000000000001000000001000000000\\
0001x0000000000001110001001010000000\\
00100x000000000100100000000000000000\\
000110x00000000001000001001000000000\\
0001100x0000000001000000001000000000\\
00000000x100000010000000000011000000\\
001011000x00000011000101001000000100\\
0001100000x0000001000001000010001000\\
10100100000x000000000000001000000000\\
000010000000x00101100000001010000000\\
0001100000000x1001000001001000001000\\
00011010000000x001000001001010000000\\
001011000000100x00100000000000000000\\
0010110001000000x0100100000001000100\\
10001000000000000x100001001000000000\\
001011000000000110x00000000000000000\\
0011110000000000011x0001001010000000\\
00011000000000001110x001001000000000\\
001011000100000011010x01001010000100\\
0101101000100110110101x1001010001100\\
10011000000000001100010x000010000000\\
000010000000100110100000x00001000000\\
0000110001000000001000000x0000000010\\
00001000000000000011000000x000000000\\
000010000000100110100000000x01000000\\
0001100000000000010000010010x0000000\\
00001100000000011010000000000x000000\\
000001000000000100100000000000x00000\\
0110101000100110100101000010100x1100\\
01111010000000000101000000101000x000\\
001111000100000011000101001010000x00\\
1000010000000000000000000000000000x0\\
00101100000000011010000001100010001x
\end{array}\right)
\qquad
23 = \left(\begin{array}{c}
x00100000000000001000001001000000000\\
0x0110000000000000000000000010000000\\
00x001000000000000100000000000000000\\
000x00000000000000110000001000000000\\
0000x0000000000100101000000000000001\\
00001x000000000000100000001000000000\\
000110x00000000001101000001000000000\\
0000100x0000000000100000000000000000\\
00000000x100000010000000000001000000\\
000000001x00000010000000000001000000\\
0000000000x0000000000000000000000000\\
00001100000x000000000000000000000010\\
000000000000x00100000000000000000001\\
0000001000000x0000000000000010000000\\
00001000000000x000001001000010000000\\
000010000000100x01101000001000100001\\
0000100000000000x0101000000011000000\\
00011000000000000x101000000000000000\\
000011000000000101x01000001000000001\\
0000101010000000111x1001001010000000\\
00011000100000001110x000001001000000\\
000010000100000011100x00001011000000\\
0001111010000001111010x1001011000001\\
00011000000000001110100x001011000000\\
000100100000100101100000x01100000001\\
0000000000000000001000000x0000000010\\
00000000000000000010100000x000000001\\
000000000000100100100000000x00000001\\
0000100000000000111010010010x1000000\\
00001000100000001010100000000x000000\\
000000000000000100000000000000x00001\\
0000001000000010000010110000100x0000\\
00001000000000001101000000001000x000\\
000010000000000010101101000011000x00\\
0000000000000000000100000010000000x0\\
00001000000000001101000000100010001x
\end{array}\right)$$

$$25 = \begin{pmatrix}
x01101000100100101100001111000000010 \\
0x1011000000100100110000110110000010 \\
10x011000000000000100000001000000010 \\
101x11000000000001110000100100000010 \\
0000x1111000100110111000111111000011 \\
10101x000001100110101000111000100011 \\
001010x11000100011110000110111000010 \\
1010110x0000100110101000111100000010 \\
00001100x100100010101000111011000010 \\
100011001x00100010100100101001000110 \\
0010100000x0000000110000010010000010 \\
10101100000x100110100000111100100011 \\
101011111101x00111111100111111100011 \\
0010100000000x0000110000010010000010 \\
00101000000000x000110000000010000010 \\
100011001001100x11101000111101100011 \\
0000110010010001x0101000011000000011 \\
10101100000000001x101001001000000010 \\
101111000000100111x01001111011100011 \\
0010110000001001101x1000110011000010 \\
00001100000000011010x000011001000010 \\
001010001100000010100x00010001000110 \\
0010100000000000001100x0010010000010 \\
10010100000000001110100x011000000010 \\
101011111101100111111000x11101100011 \\
1000110000011001001000001x0100100011 \\
10000100000000001110100100x001000010 \\
001011000001100100100000110x01100011 \\
0010110010000001101100000100x0000010 \\
00001000110000001010100001000x000010 \\
100001000000100110100000110001x00011 \\
0010100000000000001100000000100x0010 \\
00101000000000000011000000001000x010 \\
000010000100000010100100000010000x10 \\
0000000010001001001000001100000000x1 \\
00000100000110011010000000100010001x
\end{pmatrix}
\quad
26 = \begin{pmatrix}
x00001000000000000010000000100000000 \\
0x0010000000000000001000000010000000 \\
00x001000000000000000000000000000000 \\
000x10000000000000011010000100000000 \\
0000x0000000001001000000000000000001 \\
00001x000000001001000000000000000000 \\
000010x00000000001000000000000000000 \\
0000100x0000000001000000000000000000 \\
00000000x100000010000000000001000000 \\
000010000x00000010000000000000000000 \\
0000101000x0000000010000000010000000 \\
00000100000x000000000000100000000000 \\
000011000000x00101100000000001000000 \\
0000101000000x0000010000000010000000 \\
00001000000000x000000000000010000000 \\
000000000000000x00100000000000000001 \\
0000100000000001x0100000000000000001 \\
00000000000000000x101000000100000000 \\
000000000000000100x00000000000000001 \\
0000100000000000001x0000000000000000 \\
00000100000000010010x000000000000000 \\
000000000100000010000x00000000000000 \\
0000100000000000000100x0000010000000 \\
00010000010000000110000x001001000000 \\
000011000000000100100000x00000000000 \\
0000010000000000001000000x0000000000 \\
00000100000000000010100000x000000000 \\
000011000000100100000000100x00000000 \\
0000110000000000001000000000x0000000 \\
00001000000000001010000000000x000000 \\
000000000000000010010000000000x00001 \\
0000101000000000001100000000100x0000 \\
00001110000000000011000000001000x000 \\
000000000100000010000100000000000x00 \\
0000000000000001001000000000000000x1 \\
00000000000000010010000000000000000x
\end{pmatrix}$$

$$27 = \begin{pmatrix}
x00101000000000000100000001000000000 \\
0x0000000000000000000000000000000000 \\
00x001000000000000000000000000000000 \\
000x00000000000011000000010001000000 \\
0000x0000000000000100000000000000000 \\
00000x000000000000100000000000000000 \\
000110x00000000001000000001000000000 \\
0000100x0000000000000000000000000000 \\
00000000x000000010000000001001000000 \\
000000000x00000011000000001001000000 \\
0000000000x0000000000000000000000000 \\
00001000000x000000000000000000000000 \\
000000000000x00111000000001001000000 \\
0000000000000x0000000000000000000000 \\
00000000000000x000000000000000000000 \\
000000000000000x01100000001001000000 \\
0000000000000000x1100000001001000000 \\
00000000000000001x100000001001000000 \\
000010000000000011x00000001001000000 \\
0000100000000000000x0000000000000000 \\
00000000000000000000x000000000000000 \\
000000000100000010000x00001001000000 \\
0000000000000000000000x0000000000000 \\
00000000000000001110000x001001000000 \\
000000000000100111000000x01001000000 \\
0000010000000000000000000x0000000000 \\
00000000000000001100000000x001000000 \\
000000000000100100000000000x00000000 \\
0000100000000000110000000010x1000000 \\
00000000000000001110000000100x000000 \\
000000000000000010010000000000x00000 \\
0000000000000000000000000000000x0000 \\
00000000000000000000000000000000x000 \\
000000000000000000000000000000000x00 \\
0000000000000000100100000000000000x1 \\
00000000000000001001000000000000000x
\end{pmatrix}
\quad
28 = \begin{pmatrix}
x00111000000000000110000001000000000 \\
0x0010000000000000001000000010000000 \\
00x001000000000000000000000000000000 \\
000x00000000000000010000000100000000 \\
0000x0000000001001000000000000000000 \\
00001x000000000000100000000000000000 \\
000010x00000000000000000000010000000 \\
0001100x0000000001000000000100000000 \\
00000000x100000010000000000001000000 \\
000010001x00000010000000000001000000 \\
0000100000x0000000001000000010000000 \\
10100100000x000000000000010000001000 \\
000111010000x00101100000101100000000 \\
0000100000000x0000000000000000000000 \\
00001000000000x000000000000000000000 \\
000010000000000x00100000000000000001 \\
0000100011000001x0100000000001000001 \\
00011000000000000x100000000100000000 \\
000011000000000100x00000000100000001 \\
0000100000000000001x0000001010000000 \\
00001000000000010010x000000000000000 \\
000010001100000010000x00000001000000 \\
0000100000000000000100x0000010000000 \\
00001000000000000010000x001000000000 \\
000011010100000110101000x01111000000 \\
0000010000000000000000000x0000000010 \\
00011000000000000011000000x000000000 \\
000011000000100100100000101x00000000 \\
0000100000000000000100000000x0000000 \\
00001000110000000100000000000x000000 \\
000000000000100100000000100100x00000 \\
0000100000000000000000000000100x0000 \\
00001000000000000000000000000000x000 \\
000010000100000010000000000000000x00 \\
0000010000000000000000000000000000x1 \\
00000000000000011010000000000000001x
\end{pmatrix}$$

$$29 = \begin{pmatrix}
x0000100000000000000000000001000000000\\
0x0011000000000000000000000010000010\\
00x0010000000000000000000000000000010\\
000x0000000000000100000000100000010\\
0000x1000000000100100000000000000000\\
00000x0000000001101000000010000000011\\
000011x0000000001000000000001000010\\
0000110x1000000010000000001001000010\\
00000100x0000000100000000000001000010\\
000011000x0000001000000000011000010\\
0000110000x0000000010000001010000010\\
0000010000x0000000000000000000000010\\
0000011101000x0011010000100101100011\\
0000110000000x0000000000000010000010\\
0000110000000x0100000000000010000010\\
0000110000000100x1010000000100000001\\
0000110011000001x0100100000011000111\\
0001010000000000x0000000010000000010\\
0000110000000000110x0000000100100011\\
0000110000000000000x0000000010000000\\
0000000000000000000000x000000000000000\\
0000110001000000100000x00000000000000\\
0000110000000000000000x0000010000010\\
0000110000000000000000x0010000000010\\
0000110000001001000000000x00100100010\\
0000100000000001001000000x0000000011\\
0000010000000001001000000x000000010\\
0000000000001001001000000x00100000\\
0001110011001001111000000010x1000011\\
0000110011000000101000000101x100010\\
0000110000001001001000000000000x00011\\
0000110000000000000000000000100x0010\\
0000110000000001001000000001000x010\\
0000100011000001000010000001000x00\\
0000010000000001001000000000000000x1\\
0000100000000000100100000100100000x
\end{pmatrix}$$

$$30 = \begin{pmatrix}
x0010100000000000100000001000000000\\
0x0010000100000000000000000010000000\\
00x0010000000000000000000000000000000\\
000x1000010000000110000000100100000\\
0000x0000000000010101000001001000000\\
00001x0000000000101000000000001000010\\
000110x0000000000110000100100100000\\
0000100x0000000010000000001001000000\\
00000000x1000000100000000000001000000\\
000000001x0000001000000000000001000000\\
0000101000x0000000000000000010000000\\
1000010000x0000000000000010000000000\\
0000000000000x0010010000000000001000000\\
0000100000000x0000010000000010000000\\
0000100000000000x0000000000000010000000\\
0000010000000000x1010000000000001000011\\
0000100001000000x0100000001001000001\\
0001000000000000x1000000001001000000\\
0000110000000000110x0000000100100011\\
0001100010000000010x0000001011000000\\
0000000000000000001010x0000010010000000\\
0000000001000000010000x0000000000000000\\
0000100000000000000000x0000010000000000\\
0001100000000000100000x0010010000000\\
0000010001001001001000000x00001100000\\
0000010000000000000000000000x0000000010\\
0001100000000000111000000000x001000000\\
0000110000001000000000000100x00100000\\
0000100000000000001000000000x0000000\\
0000100000000000011100000000100x000000\\
0000010000001001000000000000001x00000\\
0000100000000000000000000000000100x0000\\
0000100000000000000000000000000001000x000\\
0000000001000000100001000000000000x00\\
0000000000000000100100000100000000x1\\
0000000000000000100100000000000000000x
\end{pmatrix}$$

$$31 = \begin{pmatrix}
x0000000000000000000000000001000000000\\
0x0010000000000000000000000000000000000\\
00x0010000000000000000000000000000000000\\
000x0000000000000100000001000000000\\
0000x0000000000000100000000000000000\\
00000x0000000000001000000000000000000\\
000010x0000000000000000000000000000000\\
0000100x0000000000000000000000000000000\\
00000000x0000000100000000000000000000\\
000000000x0000001000000000000000000000\\
0000100000x0000000000000000000000000000\\
0000010000x0000000000000000000000000000\\
000000000000x0010000000000000000000000\\
0000100000000x0000000000000000000000000\\
0000100000000x0000000000000000000000000\\
000000000000000x0100000000000000000000\\
0000000000000000x1000000010000000000\\
00000000000000100x0000000000000001\\
0000100000000000000x0000000000000000\\
0000000000000000010x0000000000000000\\
0000100000000000000000x00000000000000\\
0000100000000000000000x0000000000000\\
0000000000000000010000x000000000000\\
0000000000000001000000000x00000000\\
0000100000000000000000000000x0000000\\
0000000000000000100000000000x00000\\
0000100000000000000000000000000x0000\\
0000100000000000000000000000000000x000\\
0000100000000000000000000000000000000x00\\
0000000000000000010000000000000000x1\\
0000000000000000100100000000000000000x
\end{pmatrix}$$

$$32 = \begin{pmatrix}
x0000000010000000100000100100000000\\
0x0010101000000000000000000010000000\\
10x0010000000000000000000000000000000\\
100x1000010000000100000000100000000\\
0000x0000100100101100000001000000001\\
00001x0000000000001000000010000000000\\
010010x0000000000100000000101100000\\
0000100x0100000001101000001000000000\\
00000000x1000000100000000000011000000\\
000000000x0010011000100100111100000\\
0000100000x0010000010100000010001000\\
10000100000x0000000000000000000000010\\
000011000100x0010010000010110000000\\
0100101000100x1000010011000010001100\\
010010000000000x0000010010000100010000\\
000011000000100x1010000000100010000\\
000010000100100000x1000010100111010000\\
000010000100100000x1000000010110000\\
000010100100100101x0000000111110000\\
0000100000000000001x0000001010000000\\
000010010100100111101x000001010000000\\
00001000110010011110 0x01001001000100\\
010010000000001010100 0x0001010000000\\
000011000000100011111000x0010100000\\
000010000000100100110000x01000000001\\
000001000000000001000000x0000000010\\
000000000000000111100000000x000000001\\
000000000000100110100000001x01100000\\
000010000000100101100000000x0000000\\
000010000100100111100000000000x000000\\
000010000000100111100000100100x00000\\
000010000000001101000000000100x0000\\
010010100010010100110101000010000x000\\
010010001100100111101000001111000x00\\
000011000000000001100000000000000x0\\
0000000000000000100100000000000000x
\end{pmatrix}$$

$$
33 = \begin{pmatrix}
x00101000000000000110000001000000000 \\
0x0010000000000000101000000010000000 \\
00x011000000000000100000000000000000 \\
000x10000000000000110000100100000000 \\
0000x1001000100010111000001011001000 \\
00000x000000000000000000000000000000 \\
000110x100000000001001000010000000000 \\
0000100x000000000010000000000000000 \\
00001000x100000010000000000001000000 \\
000000000x000000001000000000000001 00 \\
0000101000x000000010000000001000100 0 \\
10100100000x000000000000000100000010 \\
000000000000x00000000000000000000000 \\
0000100000100x0000010000000010001000 \\
00001000000000x001011000000010001000 \\
000000000000000x00000000000000000000 \\
0000000010000000x0000100000001000100 \\
10010000000000000x101001001000000000 \\
000000000000000000x00000000000000000 \\
0000100010000000001x0001000011000000 \\
00000000000000000000x000000000000000 \\
000000000110000000000x00000001000100 \\
0100101000100010001100x0000010001000 \\
10010000000000000110000x001000000000 \\
000010000000100100000000x00100000000 \\
0000110000000000000000000x0000100010 \\
10011000000000000100100000x000000000 \\
000000000000100100000000100x00000000 \\
0001100010001000000000000000x1001000 \\
00001000110000001000000000000x000000 \\
000001000000000100000000000000x00000 \\
0000101000100010000100000000100x1000 \\
00001110000000100001000001001000x010 \\
000000000100000010000100000001000x00 \\
0000110000000000001000000100000000x0 \\
00000000000000010010000000000000000x
\end{pmatrix}
\qquad
34 = \begin{pmatrix}
x00010000000000000100000001000000000 \\
0x0010000000000000100000000010000000 \\
00x001000000000000100000000000000000 \\
000x00000000000000101001001000000000 \\
0000x000000000001101010000001000000 0 \\
00100x000000000000100000000000000000 \\
000010x00000000000100000000000000000 \\
0000100x000000000010000000000000000 0 \\
00000000x10000001010000000000100000 0 \\
000010000x00000010100000000000000000 \\
0000100000x000000010000000001000000 0 \\
00000100000x000010000000100000000000 \\
000000000000x00100100000000000000000 \\
0000100000000x000100000000001000000 0 \\
00001000000000x00010000000001000000 0 \\
000000000000000x00101000000000000001 \\
0000000000000000x0101000000001000000 \\
00000000000000000x100000001000000000 \\
000010000000000110x000000000100000 01 \\
0000100000000000001x0000000000000000 \\
00001100000000011010x000000000000000 \\
000000000100000010100x00000000000000 \\
0000100000000000001000x0000000000000 \\
00001000000000000010100x001000000000 \\
000010000000100110000000x00000000000 \\
0000010000000000001000000x0000000010 \\
00000000000000000010100100x000000000 \\
000000000000100100100000000x00000000 \\
00001000000000000010100000 00x0000000 \\
00000000000000000010100000000x000000 \\
000000000000000000100100000000x00011 \\
0000100000000000001000000000100x0000 \\
00001000000000000010000000001000x000 \\
000000000100000010100000000000000x00 \\
0000000000000000001001000000000000x1 \\
00000000000000000010010000000000000x
\end{pmatrix}
$$

$$
36 = \begin{pmatrix}
x00100000000000000110000001000000000 \\
0x0010100000000000000000000010000000 \\
00x001000000000000000000000000000000 \\
000x00000000000000100100001000000000 \\
0000x0100000000011011100001001000000 \\
00001x000000000011010000000000000011 \\
000010x000001000001110000000100000 00 \\
0000100x000000001101000001000000000 0 \\
00001000x100000110100000000011000000 \\
000000000x000000101000000000010000 00 \\
0000101000x000000010000000001000000 0 \\
00000100000x100000000001000000000010 \\
000010000000x00110100000100000100001 \\
0000101000000x000010000000001000000 0 \\
00001010000000x00010000000001000000 0 \\
000011000000100x10100000000000000001 \\
0000110001000001x01000000000010000 01 \\
00000000000000000x100000001000000000 \\
000011000000000110x000000000100000 01 \\
0000101000000001001x000000001000000 0 \\
00001000000000001010x000000000000000 \\
000010000000000010101x00000000000000 \\
0000101000000000000100x0000010000000 \\
00000000000000000110000x001000000000 \\
000011000000100100100000x10000100001 \\
0000010000000001000000000x0000000011 \\
00001000000000000010100000x000000000 \\
000000000000000000100000100x00000001 \\
0000101000000000000100000000x0000000 \\
00001000000000011010000000000x000000 \\
000000000000000100100000000000x00001 \\
0000101000000000000100000000100x0000 \\
00001010000000000001000000001000x000 \\
000010100000000000010000000010000x00 \\
0000000000000001001000000000000000x1 \\
00001100000000011010000000000000001x
\end{pmatrix}
$$