# 5

# Graph Theoretical Dimensions of Informal Organizations

David Krackhardt
*Carnegie Mellon University*

In his classic work on the architecture of complexity, Simon (1981) noted the uncanny presence of hierarchy in virtually all complex systems. He further argued that there was a universal function to such hierarchical forms: They are efficient and robust against disruptions that might threaten the cybernetic goals of the system. And although formal organizational charts are obviously hierarchical, he argued that informal organizations also would be found to be hierarchically structured: "If we make a chart of social interactions, of who talks to whom, the clusters of dense interaction in the chart will identify a rather well-defined hierarchic structure. The groupings in this structure may be defined operationally by some measure of frequency of interaction in this sociometric matrix" (Simon, 1981, p. 197).

This idea that informal organizations will naturally evolve into a hierarchical structure is intriguing and has intuitive appeal. The theme can be found with empirical support elsewhere. For example, Michels (1915) noted that even democratically based voluntary organizations evolve toward a centralized, hierarchical structure as they grow. Guetzkow and Simon (1955) discovered that small groups that are allowed unlimited choice of communication channels tend to centralize their communication flows into a hierarchical "wheel" structure.

The normative part of Simon's claim, that hierarchy exists because it allows the system to operate more efficiently and survive outside disturbances, is also appealing. From this, we may deduce a hypothesis about the structure of informal organizations and performance.

Simon's model of hierarchy raises three unresolved issues, two theoretical and one methodological. First, as Simon admitted himself (1981, p. 213), people do communicate outside the preferred boundaries defined by the formal hierarchy of the organization. That is, perfectly hierarchical informal organizations are rare. Nonetheless, he argued, these exceptional communication links are relatively limited and not consequential to the overall pattern of hierarchy in the organization.

The second theoretical problem with Simon's model is that it flies in the face of several normative theories of organizational structure that emphasize the value of communication and information flows outside the normal, hierarchical boundaries. Burns and Stalker (1961) argued that when an organization is faced with a dynamic environment, an organic, nonhierarchical, informal structure is more appropriate for organizational effectiveness and survival. Allen (1977) demonstrated that research and development organizations can enhance their effectiveness by promoting communication outside the formal, hierarchical boundaries:

> Increased communication between R&D projects and other elements of the laboratory staff were in every case strongly related to project performance. Moreover, it appears that interaction outside the project is most important. On complex projects, the inner team cannot sustain itself and work effectively without constantly importing new information from the outside world. (pp. 122–123)

And Krackhardt and Stern (1988) presented experimental evidence that under some conditions organizations are better off if they maximize strong cross-departmental relationships.

The third problem with Simon's model is one of measurement and consequent testability. Given that pure hierarchies do not exist, we must somehow differentiate between structures as being more or less hierarchical. Otherwise, we have no way of confirming or disconfirming his predictions. He offered no specifics for measuring the degree of hierarchy in social systems. In fact, systems whose elements appear to have no hierarchical structure (systems he calls *flat* hierarchies), he argued, are still hierarchical with an indefinitely large "span."

The purpose of this chapter is to broaden Simon's ideas of hierarchical structures as they pertain to informal organizations. First, I dispense with the assertion that informal organizations are necessarily hierarchical. Instead, I argue that this is an empirical question and an appropriate object of research. Second, I propose that hierarchical forms will have implications for organizations, but that some of the implications may not necessarily enhance the organization's efficiency or ability to survive (i.e., they are not necessarily functional). Finally, I specify a method for measuring the degree to which the informal organization is structured in a way that Simon would call hierarchical. Because the ability to answer the former research questions depends on this last measurement issue, I begin with the development of measures of informal structure.

## GRAPH THEORY AND MEASURES OF STRUCTURE

Graph theory (Harary, 1969) provides us with a precise language for representing structures of all forms, including the structures Simon referred to as hierarchical. Because I draw on graph theory in this chapter, I provide definitions as necessary for clarity. A *graph* (G) is defined as a set of $N$ points P = {$P_i$} and a set of unordered pairs of those points L = {$P_i$, $P_j$}; these latter elements are often referred to as lines connecting those points. In the immediate context, these points represent people in the organization, and the pairs of points represent relationships (such as interaction, communication) between those organizational members. For example, if person $i$ interacts with person $j$, then the ordered pair ($P_i$, $P_j$) is included in set L that defines the relationship *interaction.*

A directed graph, or *digraph* (D), is defined as a set of points P = {$P_i$} and a set of *ordered* pairs of points L = {$P_i$, $P_j$}. A digraph is used to represent relations that are potentially asymmetric, such as authority or giving advice. For example, if $i$ is the immediate supervisor to $j$, and L is defined as the set of formal authority relationships, then L would contain the ordered pair ($P_i$, $P_j$) but would not contain the ordered pair ($P_j$, $P_i$).

Graph theory definitions are often easier to convey by example. Figure 5.1 provides an example of two graphs and one digraph. Graphs are represented with points and lines connecting the points. Digraphs are represented by points and lines with arrowheads on them to indicate the order of the pair of points being connected. The graph in Fig. 5.1c represents a special function. It is the *underlying graph* of the digraph in Fig. 5.1b. The underlying graph of a digraph is the graph obtained by "removing the arrows" from the digraph. That is, if digraph D contains either the ordered pair ($P_i$, $P_j$) or ($P_j$, $P_i$), then the underlying graph G will include the unordered pair ($P_i$, $P_j$).

A point and a line are *incident* with one another if the line contains the point in its pair. In Fig. 5.1a, $P_2$ is incident with line ($P_1$, $P_2$). A *walk* is an ordered sequence of alternating points and lines, starting and ending with points, such that each line is incident with the point that precedes it and with the point that follows it. A path is a walk with no repeating points. In Fig. 5.1c, the sequence $P_3$, ($P_3$, $P_5$), $R_5$, ($P_5$, $P_4$), $P_4$ constitutes a path from $P_3$ to $P_4$. One point is said to be able to reach another if there exists a path that

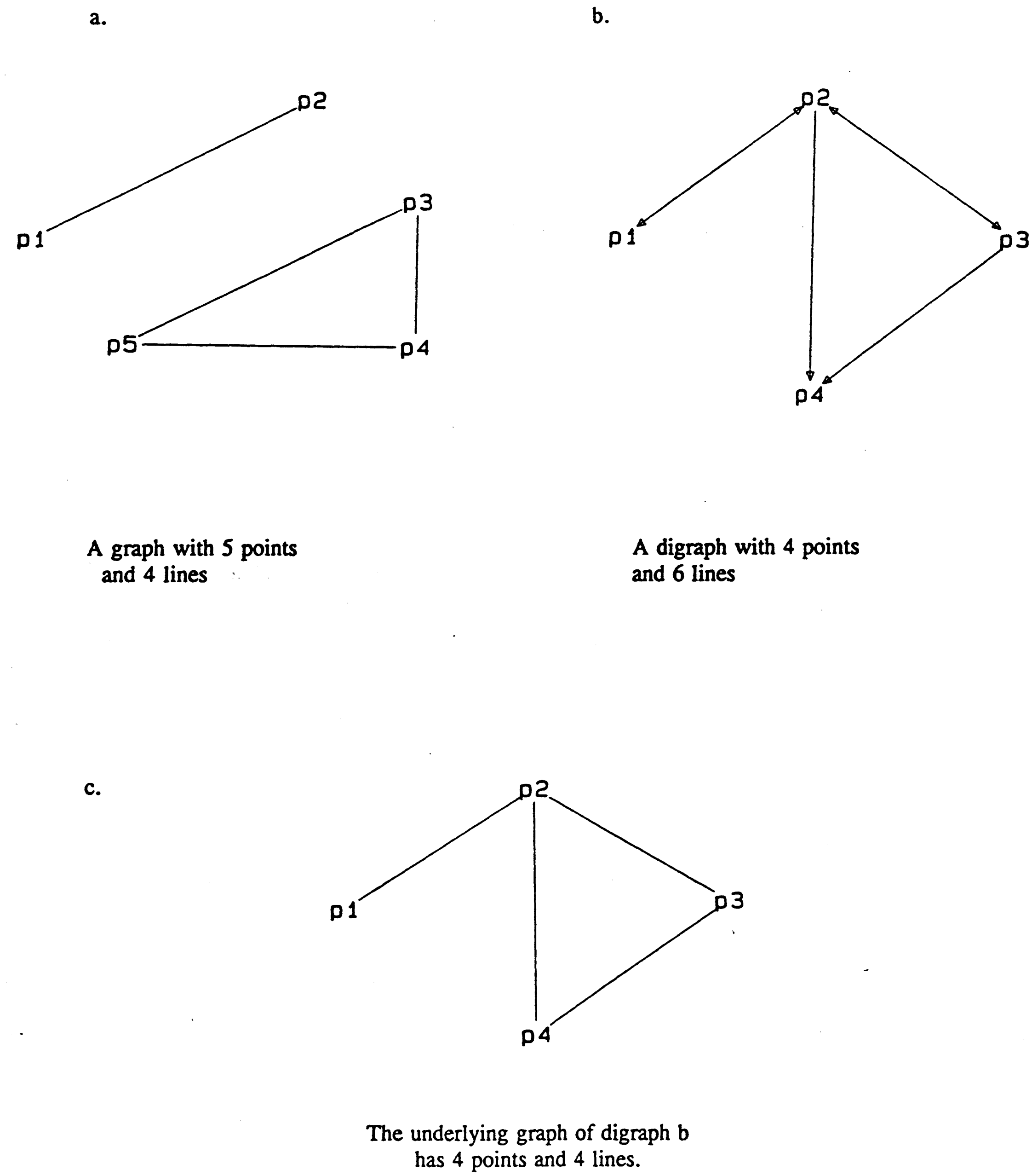


FIG. 5.1. Examples of graphs and digraphs.

starts at the first point and ends at the second. All pairs of points in the graph in Fig. 5.lc are mutually reachable.

With a small restriction in the definition of incidence of points and lines, these same definitions apply to digraphs as well. In an ordered point-line pair, $[P_i, (P_j, P_k)]$, the point and line are incident with each other if $P_j = P_i$. In an ordered line-point pair, $[(P_j, P_k), P_i]$, the line and point are incident with each other if $P_k = P_i$. The definitions of paths and reachability are identical to those in graphs. In the digraph represented in Fig. 5.1b, a path exists from $P_1$ to $P_4$ but not from $P_4$ to $P_1$. Therefore, $P_1$

can reach $P_4$ but not vice versa. In fact, $P_4$ cannot reach any other point, but each other point can reach $P_4$.

A *connected graph* is a graph in which each point can reach every other point. Figure 5.1c is connected; Fig. 5.la is not. A subgraph (S) of graph (G) is a graph whose points and lines are also in G. A component (C) of graph (G) is a connected subgraph of G with two characteristics: (a) All the lines in G incident to every point in C are included in C, and (b) there is no point in G not included in C that, in G, can reach a point included in C. Figure 5.1c has one only component; Fig. 5.la has two components.

A *connected digraph* is a digraph in which each point can reach every other point in the underlying graph of the digraph. Each point in the digraph of Fig. 5.lb is reachable from every other point in the underlying graph Fig. 5.1c. Thus, the digraph in Fig. 5.lb is connected. A *component* (C) of a digraph (D) is a connected subgraph of D with the following characteristics: (a) All the lines in D incident to every point in C are included in C, and (b) there is no point in D not included in C that, in the underlying graph of D, can reach a point included in C. The digraph in Fig. 5.lb has one component only.

With these tools and definitions developed so far, it is possible to represent the informal structure of any organization. But to fully explore Simon's notions of hierarchical structure, it will be necessary to develop some additional operations. From these, one can determine the extent to which an informal structure approximates a pure hierarchical structure.

## PURE HIERARCHICAL STRUCTURES: THE OUTTREE

The first task before us is to establish a pure structure as a standard against which other structures can be compared. For the purposes of this analysis, the ideal candidate for such a structure is, in graph theory terms, the *outtree*. Before a formal definition is presented, an intuition of what constitutes an outtree is provided for the reader in Fig. 5.2, which contains four examples. First, it should be noted that outtrees are digraphs. Second, every point, with the exception of the one point at the "top" of the outtree, has exactly one arrow pointing to it, although the points may have several arrows emanating from them. If these arrows represented authority relationships, then we might interpret this statement as noting that each point has one and only one "boss," but each point may have any number of subordinates. In fact, it should be immediately apparent to the reader that each of these figures could be examples of organizational charts—the *archetypical formal hierarchy,* as Simon termed it (1981, p. 197).

There are several reasons that the outtree serves as a reference base for

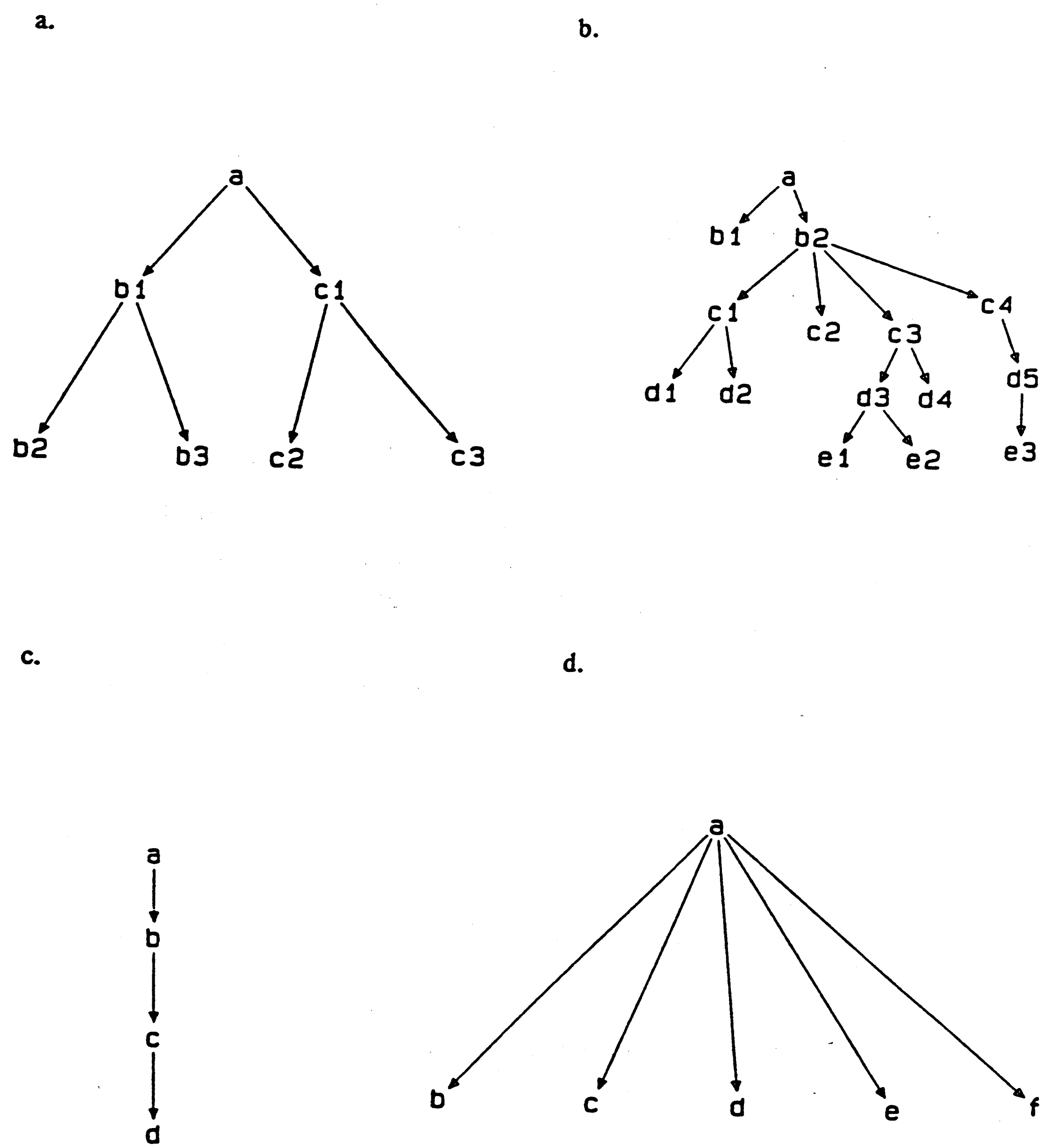


FIG. 5.2. Examples of outtrees.

the study of informal structures. First, all of Simon's hierarchical systems can be represented as an outtree.[1] Second, as mentioned earlier, they correspond to our intuition of the archetypical hierarchy, the formal organization. Third, they preserve several fundamental principles of classic organizational structure, including unity of command, unambiguous chain of command, and the scalar principle.

But this graph theory concept provides more than an archetype. It provides a basis for describing observed organizational structures and measuring their deviance from this archetype. To pursue this, it is first

[1]It is worth noting at this point that most of Simon's hierarchical systems are represented by an inclusion relation, rather than the type of interpersonal relations used throughout this chapter.

necessary to formally define an outtree using more graph theory. There are many ways to so define an outtree (e.g., Wilson, 1979, p. 45). For reasons that become clear shortly, I use the following four conditions of a digraph as a definition of an outtree. These conditions are both necessary and sufficient for the digraph to be an outtree:

1. The digraph is connected.
2. The digraph is graph hierarchic.
3. The digraph is graph efficient.
4. Every pair of points in the digraph has a least upper bound.

If a graph violates any of these four conditions, then it is not an outtree. Moreover, we can count the number of violations in each of the dimensions to give us a measure of distance from the archetypal structure. Because these violations are based on independent criteria, the picture of the structure described by each dimension differs considerably. Figure 5.3 displays some examples of these differences. In the center of the figure is an outtree. Each of the other four figures surrounding the outtree represents an extreme case where one (and only one) of each of the four conditions is violated to the maximum extent possible. It is useful to refer to this figure as each of the four dimensions is defined next.

Each of the four measures of degree of structure is based on the number of outtree violations that exist in any particular structural arrangement. As such, each condition becomes a *dimension* of structure, continuously varying in value from 0 to 1. That a graph has a value of 1.0 on all four dimensions is equivalent to stating that a graph is an outtree. Also, each of the four dimensions has different implications for the organization. Each of these dimensions and its implications for the organization is next described in turn.

*1. Connectedness.* The definition of connectedness was already provided earlier: A digraph is connected if each point can reach every other point in the underlying graph. To say that a digraph is *disconnected* implies that there are at least two components in the digraph. The degree to which the digraph is disconnected is a function of the number of violations of the connectedness condition. A violation is defined as a point being unable to reach another point in the underlying graph. If we divide the number of violations by the maximum number of possible violations (i.e., in the case where no point can reach any other point), we have a continuum representing the degree to which the graph is disconnected. Subtracting this ratio from 1 gives us the degree of connectedness in the structure. The *degree of connectedness* is then defined as:

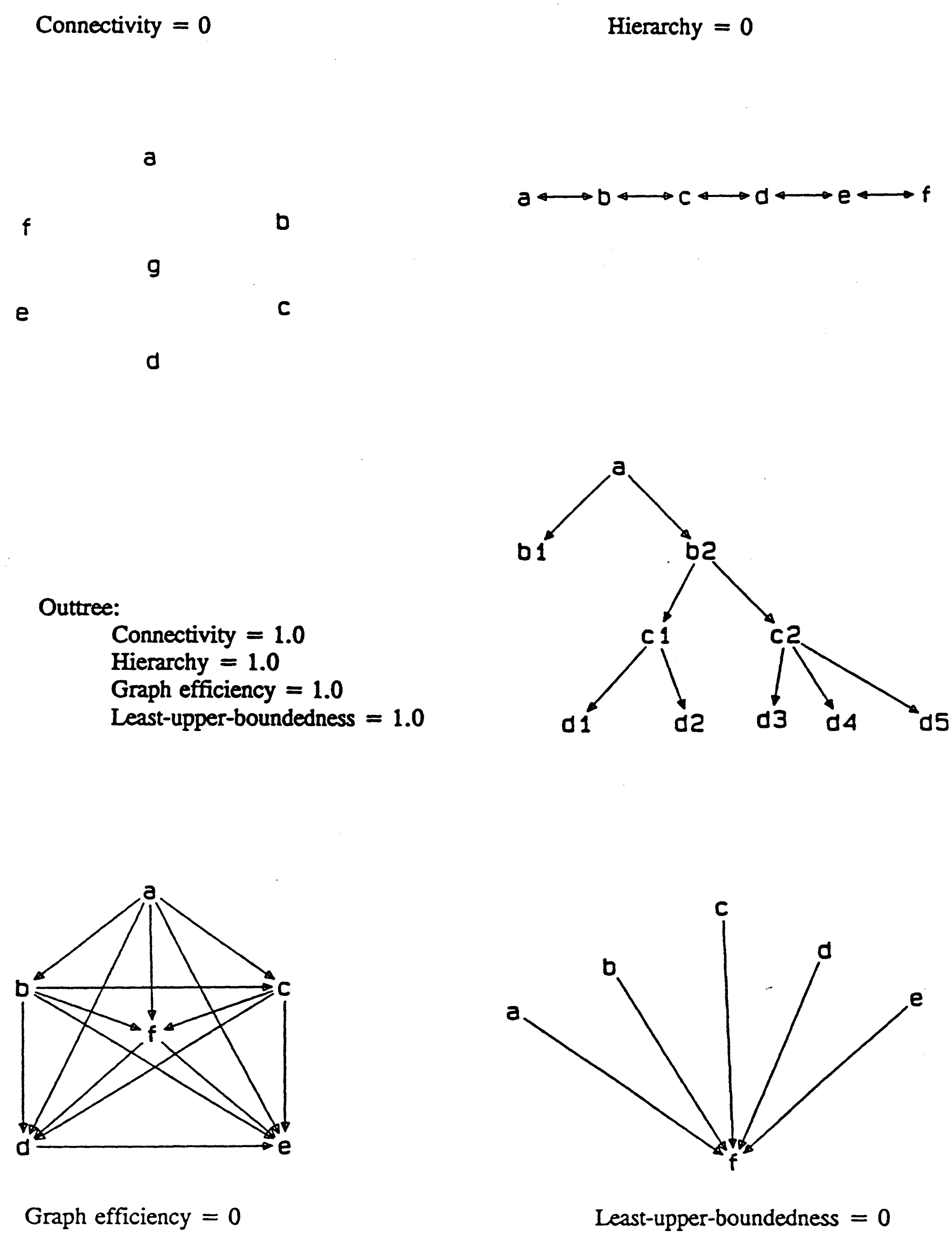


FIG. 5.3. The four dimensions of structure.

$$\text{Connectedness} = 1 - \left[\frac{V}{N(N-1)/2}\right]$$

where $V$ is the number of pairs of points that are not mutually reachable, and the maximum number of violations is the total number of pairs of points $= N(N-1)/2$.

The degree of connectedness in a set of social relations is the simplest of the measures. At one end of the spectrum, an outtree is completely connected. A disconnected graph represents a division in the social system. The more people are separated from each other, the more difficult it is to organize them through the network. At the extreme, no one is connected to anyone (connectedness = 0); everyone is an independent actor.

If the task facing the organization is routine, and the environment does not change, then connectedness may not be essential to the performance of the organization in its task. But if the organization has many exceptions that require consultation, a set of established communication and advice relations that incorporates all actors, at least indirectly, would be essential. Also, lack of connectedness may be a reflection of a major political division, such that one side does not talk to the other side(s).

*2. Graph Hierarchy.*[2] The ***graph hierarchy*** condition states that in a digraph D, for each pair of points where one ($P_i$) can reach another ($P_j$), the second ($P_j$) cannot reach the first ($P_i$). For example, in a formal organizational chart, a high-level employee can "reach" through the chain of command her subordinate's subordinate. If the formal organization is working properly, this lower level employee cannot simultaneously "reach" (i.e., cannot be the boss of a boss of) the higher level employee.

To measure the degree of hierarchy of digraph D, a new digraph $D_r$ must be created. $D_r$ is defined as the *reachability digraph* of D. Each point in D exists in $D_r$; moreover, the line ($P_i$, $P_j$) exists in $D_r$ if and only if $P_i$ can reach $P_j$ in D. If D is graph hierarchic, then $D_r$ will have no symmetric lines in it. That is, if the line ($P_i$, $P_j$) exists in $D_r$ then the line ($P_j$, $P_i$) does not. A violation to this condition exists every time a symmetric line exists in $D_r$. The degree of hierarchy, then, is defined as:

$$\text{Graph hierarchy} = 1 - \left[\frac{V}{\text{Max}\,V}\right]$$

where $V$ is the number of unordered pairs of points in $D_r$ that are symmetrically linked (that is, where $P_i$ is linked to $P_j$ and $P_j$ is linked to $P_i$), and Max$V$ is the number of unordered pairs of points in $D_r$ where $P_i$ is linked to $P_j$ or $P_j$ is linked to $P_i$.

Graph hierarchy exists to the extent that the relations are strictly ordered. For example, hierarchy occurs if relations are determined by status, prestige, or formal authority. Informal relations, such as advice relations, can be ordered, but are not necessarily so. An outtree (such as the organizational chart) is perfectly hierarchical. At the other extreme, if there

[2]The term ***hierarchy*** is used differently here than by Simon. Nothing in Simon's work specifies asymmetry of relations. However, the term's use in this chapter corresponds more closely to the common use of the word. From here on, I will use the term ***hierarchy*** to refer to graph hierarchy rather than Simon's definition.

is no status in a system, then no graph hierarchy is likely to emerge in the informal relations.

A mechanistic organization is likely to be very status ridden (Shrader, Lincoln, & Hoffman, 1989). Members are likely to go up the organization for advice. To the extent that this is true, a mechanistic organization will be characterized by a high degree of hierarchy in advice relations. In an organic organization, on the other hand, status is more diffuse, and project leaders may not hesitate to seek advice from subordinates or someone from a different group in the organization. In such an environment, advice relations might not be hierarchically arranged at all.

*3. Graph Efficiency.* One of the conditions of an outtree is that the underlying graph is connected and contains exactly $N-1$ lines. Fewer lines than that and the digraph disconnects into components. More lines than that creates multiple paths and cycles between points. In a sense, these multiple paths are redundant in graph-theory terms, and they disrupt the stoic, bare-bones nature of the pure outtree structure.

The technical definition of the *graph efficiency* condition is: In the underlying graph ($G_1$, $G_2$, etc.) of each component ($D_1$, $D_2$, etc.) of digraph D, there are exactly $N_n-1$ links, where $N_n$ is the number of nodes in the corresponding component $D_n$. Because fewer than $N_n-1$ links is not possible (because that would break the component into subcomponents), violations occur to the extent that more than this minimum number of links is present. The degree of graph efficiency is defined as:

$$\text{Graph efficiency} = 1 - \left[\frac{V}{\text{Max}\,V}\right]$$

where $V$ is the number of links in excess of $N_n-1$, summed over all components, and Max $V$ is the maximum number of links in excess of $N_n-1$ possible, summed over all components.

Links are not without costs in a social system. They take time and resources to maintain. Thus, the concept of graph efficiency characterizes how dense the network is beyond that barely needed to keep the social group even indirectly connected to one another.

Graph inefficiency should not be confused with social inefficiency or economic inefficiency. To say that a group is graph inefficient simply implies it has more than the $N-1$ links required to remain connected. In fact, in a high-tech, organic organization faced with a dynamic and unpredictable environment, graph inefficiencies may be called for to facilitate the quick cross-fertilization of innovative ideas (Shrader et al., 1989). Thus, graph efficiency reflects the cost of a dense network; it avoids answering the question about the benefits of such a network.

Nonetheless, some conjectures could be made about the relationship between graph efficiency and organizational efficiency. An organization that is so bare bones in its informal structure that it is perfectly graph efficient is also fragile. It is vulnerable to the arbitrary deletion of a link or point (for example, through attrition). Some redundancies (graph inefficiencies) in the informal network also help to short-circuit long communication paths and thus that may slow down information flow. Thus, very high values of graph efficiency are likely to be associated with less than optimal organizational performance.

On the other hand, extremely dense informal networks that would characterize very low efficiency scores are likely to be overburdened with networking. Employees cannot be expected to relate to everyone else in the organization. People would spend all their time interacting and have little left over for getting their work done. Thus, we expect a curvilinear relationship between graph efficiency and organizational effectiveness, with the optimum graph efficiency value to lie between 0 and 1.

*4. Least Upper Boundedness.* In order for a pair of actors to have a *least upper bound* (LUB), they each must have access to a common third person in the organization to whom they both can "appeal" (through the network). This third person (called an *upper bound*) must be someone to whom they both defer (either directly or indirectly) in the network. A given pair of actors may have many upper bounds. In such cases, a least upper bound is a member of that set of upper bounds who in turn can appeal to or defer to the remaining upper bounds. In a formal organization chart, the LUB of two employees is the closest boss who has formal authority over both of them.

The technical definition of the least upper bound condition is given as follows: Within each component ($D_1$, $D_2$, etc.) of digraph D, each pair of points ($P_i$ and $P_j$) has at least one least upper bound (LUB). An upper bound for a pair of points ($P_i$ and $P_j$) is a third point ($P_k$) from which there is a path to each of the pair; a least upper bound is an upper bound ($P_k$) that is included in at least one directed path from each other upper bound ($P_l$, $P_m$, . . .) to each of the pair ($P_i$, $P_j$). Violations to this condition occur whenever a ($P_i$, $P_j$) pair of points in $D_n$ has no LUB. The degree of LUBedness is defined as:

$$\text{LUB} = 1 - \left[\frac{V}{\text{Max}\,V}\right]$$

where $V$ is the number of pairs of points that have no LUB in each component summed across all components, and Max $V$ is the maximum number of pairs of points that could possibly have no LUB. It should be noted that point $k$ may be equal to point $i$ or $j$. That is, two points ($P_i$ and $P_j$) that are connected always have as a LUB one of the two points ($P_i$ or $P_j$ or both). Because a component $C_n$ must have at least $N_n - 1$ lines, every component has by definition at least $N_n - 1$ pairs of points that do have LUBs. Therefore, the maximum number of violations for a component of a digraph is:

$$\text{Max}\,V = \frac{(N_n - 1)\ (N_n - 2)}{2}$$

This is the most complex measure of the four measures of structure. It has interesting implications for structures, however. First, it is the only condition of the four that is sensitive to the direction of the arrows in the digraph. It is possible to change the LUB score from 0.0 to 1.0 (or vice versa) simply by changing the direction of all of the arrows in a digraph. Thus, the meaning of the direction of a relationship becomes important here. It is assumed that all relationships are defined in a way that suggests that an arrow from $P_i$ to $P_j$ implies that $P_j$ defers to $P_i$, or that $P_i$ has more status than $P_j$.

The LUB condition preserves the unity-of-command principle in formal organizations. It also ensures that there is only one "chief executive" at the top. Violations of the condition are an indication that there may be too many informal cooks spoiling the pot. For example, in Fig. 5.3, the digraph in the lower right corner has a LUB score of 0, indicating that the number of violations is at a maximum. If the relationship represented by the arrows happened to be authority, the lower point in the digraph would be subject to the orders of five different "top dogs."

It has been suggested elsewhere (Doreian, 1971; Friedell, 1967) that a LUB condition in an informal network is an indication of how differences or conflict might be managed within the organization. If a LUB exists for a pair of actors, then that LUB person has a potential position for settling or dealing with the conflict. When relatively few of the pairs of actors have a LUB, then conflict would be predicted to be difficult to resolve in the organization.

## BEHAVIOR OF THE FOUR DIMENSIONS IN RANDOM DIGRAPHS

Thus far, four measures of structure have been proposed along with tentative relationships to organizational phenomena. We are left with the empirical question, what do organizations look like in the real world? But before that question is pursued, there is another empirical question worth exploring: How do these measures behave in structures that are truly random rather than ordered, as the word structure implies? The answer to this question provides the empiricist with a type of null hypothesis against which he or she can compare real-world observations.

To study this question, random structures were generated. Two parameters were manipulated in these random digraphs, the number of points in the digraph (= *N*) and the probability that any two points are connected to each other with a line (= *P*). Four different sized digraphs were created: *N* = 5, 10, 20, and 50. The value of *P*, which virtually determines the number of lines in the digraph, varied from .01 to .10 in increments of .01, and then from .15 to .9 in increments of .05. For each combination of *N* and *P*, 500 digraphs were generated, for a total of 52,000 digraphs. For each digraph generated, the degree of connectedness, graph hierarchy, graph efficiency, and least-upper-boundedness was calculated.

The results of these simulations are depicted in the Appendix. In each graph the mean value, the 95th percentile value, and the 5th percentile value of one of the four structure measures are plotted as a function of *P* for the given *N*.

These plots provide ranges of values one would expect if the lines of the digraphs were randomly drawn. Two general conclusions are evident from these simulations. First, the range of probable structure values for a given *P* is greatly reduced as *N* increases. Second, the relationship between *P* and each of the specific measures is very strong in random graphs when *N* is large (= 50).

The fact that *P*, and by implication the density of the relations, is so closely correlated with these graph values in large digraphs is not a surprise. For example, perfect connectedness is impossible until the digraph has at least $N-1$ lines. Beyond that, it is expected that the more lines there are the easier it would be to randomly create a connected digraph. And graph efficiency is easily construed as a surrogate for density; thus, the near linear relationship between *P* and efficiency when $N = 50$ is no surprise.

Two points are worth underscoring in relation to these simulation results. First, just because values of these structural dimensions in random digraphs seem heavily constrained by *P* and *N* does not mean that real-world digraphs cannot be found outside these values. For example, the digraph in the lower left corner of Fig. 5.3 (with a graph efficiency of 0) is a kind of structure that has $P = .5$ and a hierarchy of 1.0. This structural pattern could exist for any *N* size. If the graph were size $N = 10$, the simulation results indicate that the graph hierarchy of 1.0 would be incredible, if not virtually impossible. Of course, this case is quite possible in the real world, where status often aligns such deference relations just as a magnet aligns free-floating iron particles in its field. Observing such a structure, we would conclude that there was such a force aligning these relations, because we would never expect to see such a structure by chance. These simulation results, then provide a reference point for comparison as real-world data are collected.

The second point is a cautionary note. These four structural measures are sensitive to densities. And densities in relations are very sensitive to how the relation is measured. We often collect such data by asking employees directly whom they talk to, whom they go to for advice, whom they seek out in the case of problems, and so forth. Sometimes we modify these questions with temporal ranges, such as ". . . at least every day," ". . . at least once a week." Such modifications can greatly affect densities. Even when the same question is consistently asked, the researcher can affect the density of the relation by insisting on various levels of confirmation of the relation (e.g., both actors must agree that $P_i$ goes to $P_j$ for advice before the $P_i$, $P_j$ line is drawn). These simulations suggest that researchers wishing to make cross-organizational comparisons along these four dimensions must pay particular attention to how the method of data collection could affect the variance in density among the organizations studied.

## CONCLUSION

In this chapter I have argued that graph theory provides a rich descriptive language for assessing organizational structure. With few exceptions (e.g., Mackenzie, 1986; Shrader et al., 1989), little research has taken advantage of this natural link between mathematics and organizational theory.

But not only does graph theory describe, it permits us to measure the degree of structure in organizations in more precise terms than is ordinarily done in field research. Moreover, by dividing the ultimate structure, the outtree, into its four constituent parts, different qualities of lack of structure can be measured. It is suggested that these four dimensions are each related to different organizational phenomena:

1. Connectedness is associated with the ease with which the organization can deal with and implement change.
2. Graph hierarchy is associated with the degree to which the organization is dominated by status in its informal relations.
3. Graph efficiency has a curvilinear relationship to organizational effectiveness.
4. Least-upper-boundedness is associated with organizational conflict.

Empirical evidence in support of these conjectures awaits us. In due course, I expect these predictions to be modified, conditioned, and perhaps even discarded and replaced with more accurate theories. In the meantime, using these graph-theoretic concepts to build on Simon's original idea of hierarchy should allow us to better grasp the role of organizational structure as an independent variable in organizational theory.

# APPENDIX
# STRUCTURE OF RANDOM GRAPHS BY *P*

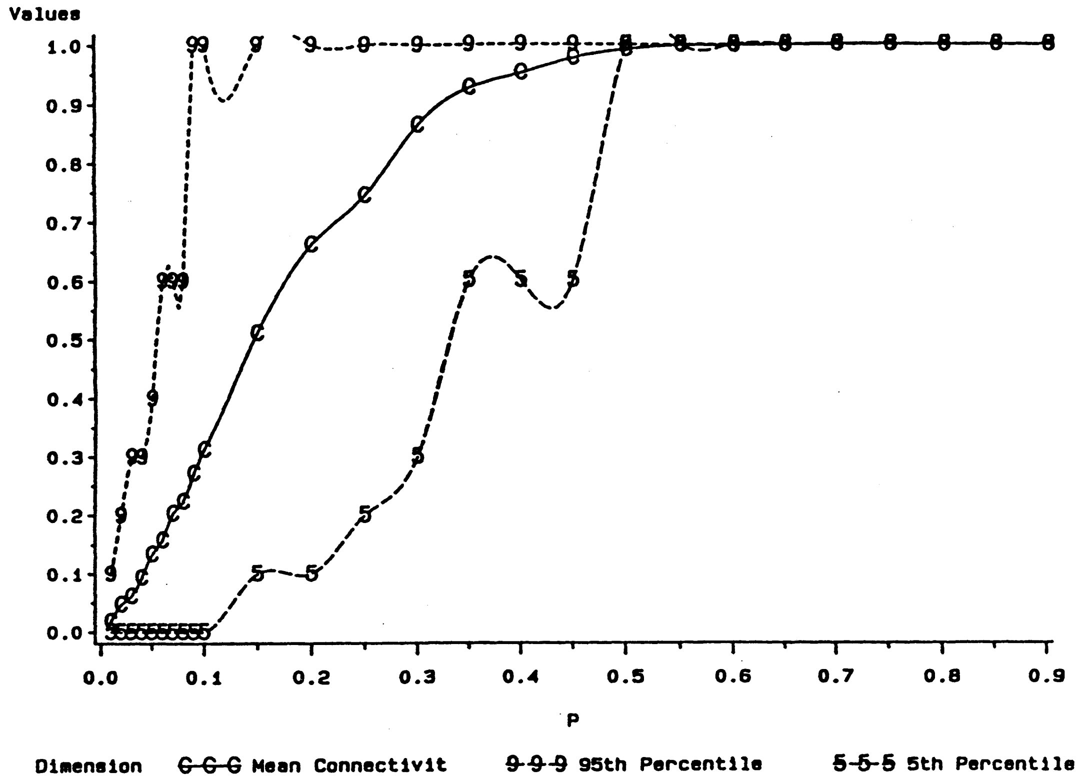


FIG. 5.A1. N=5.

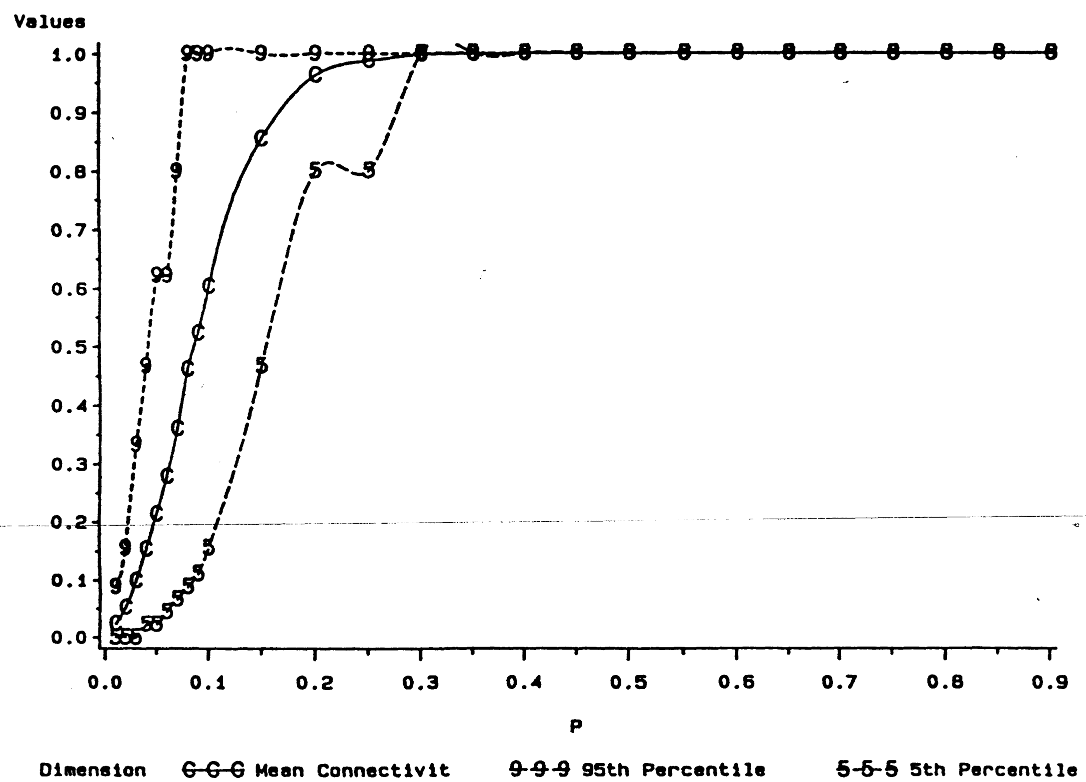


FIG. 5.A2. N=10.

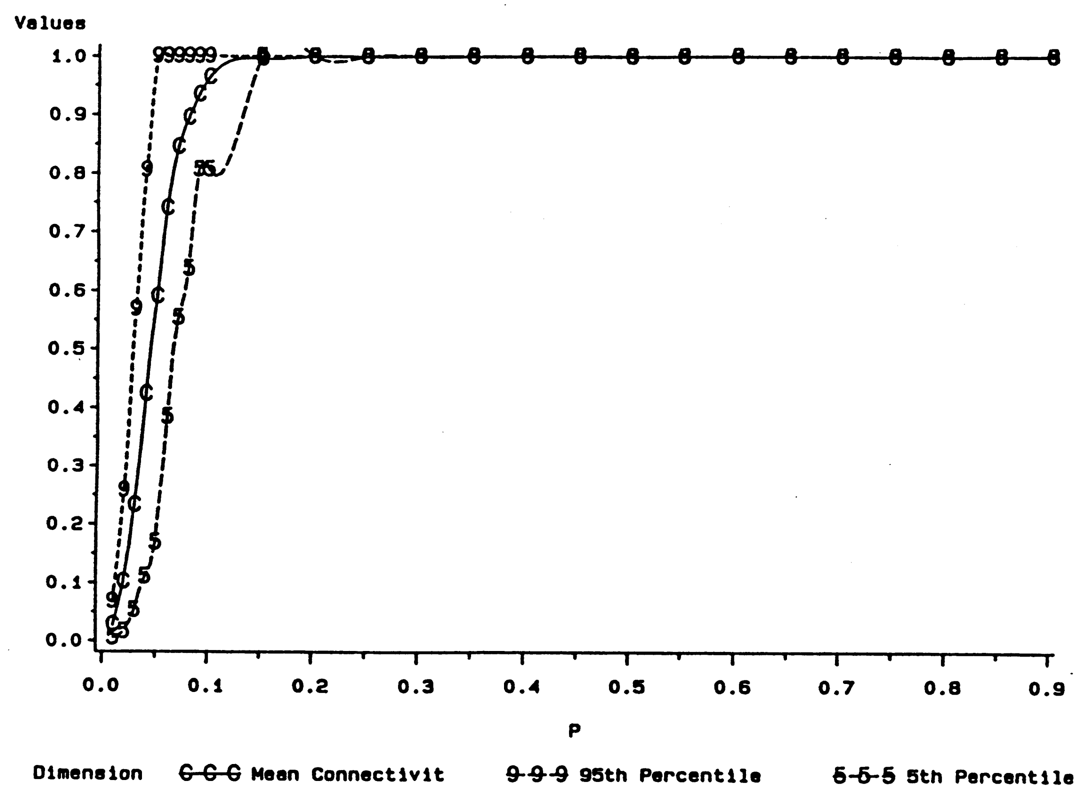


FIG. 5.A3. N=20.

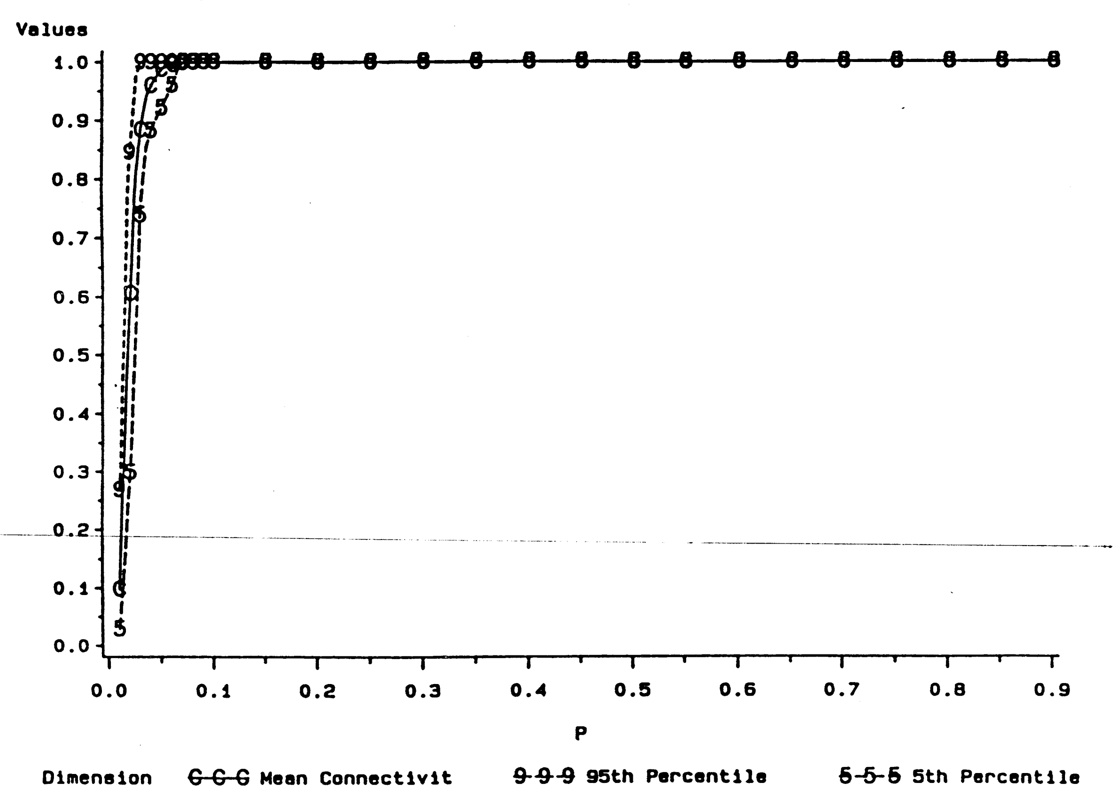


FIG. 5.A4. N=50.

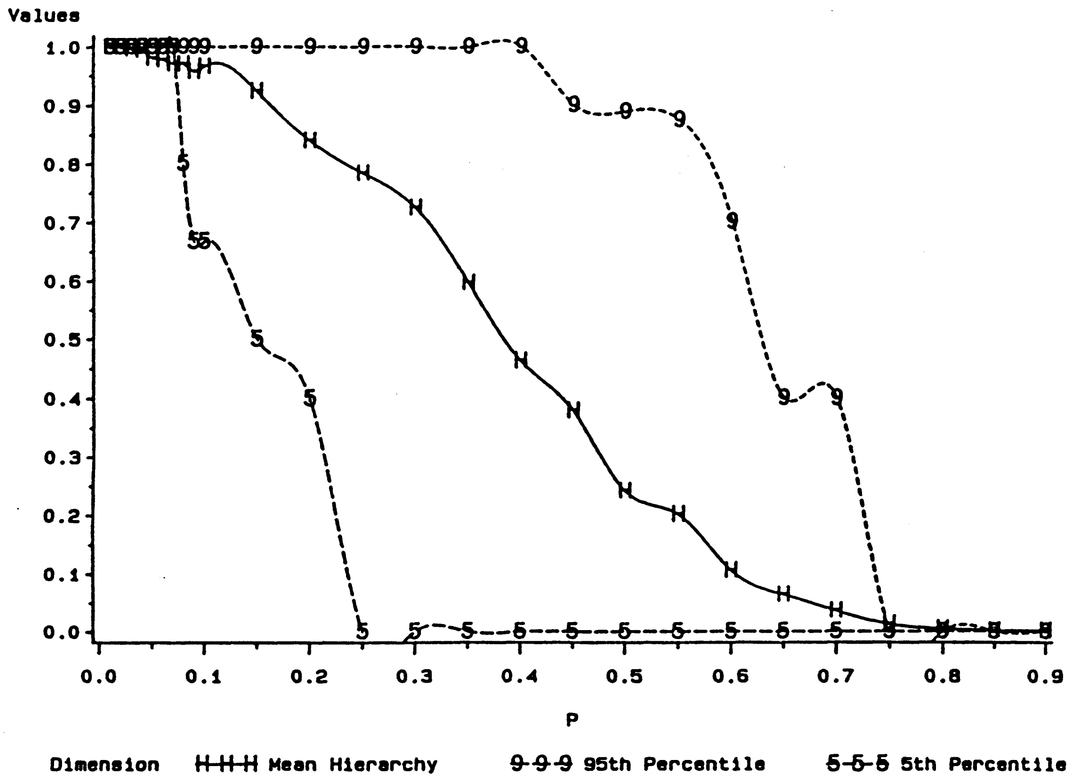


FIG. 5.A5. N=5.

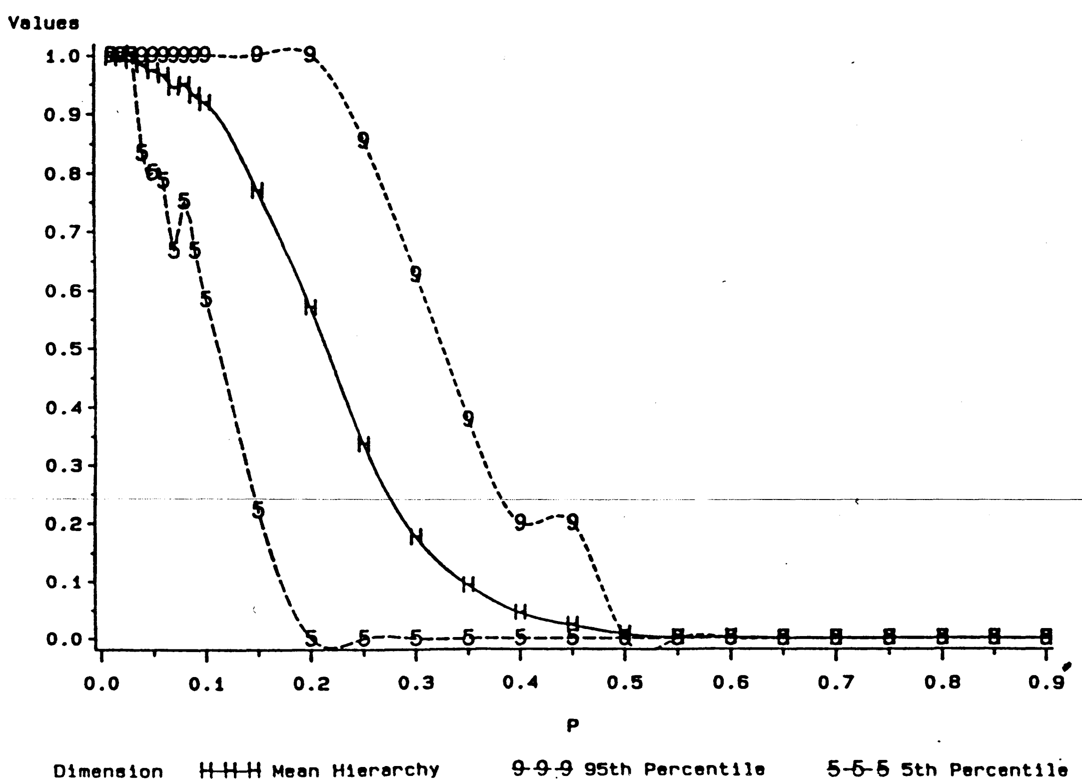


FIG. 5.A6. N=10.

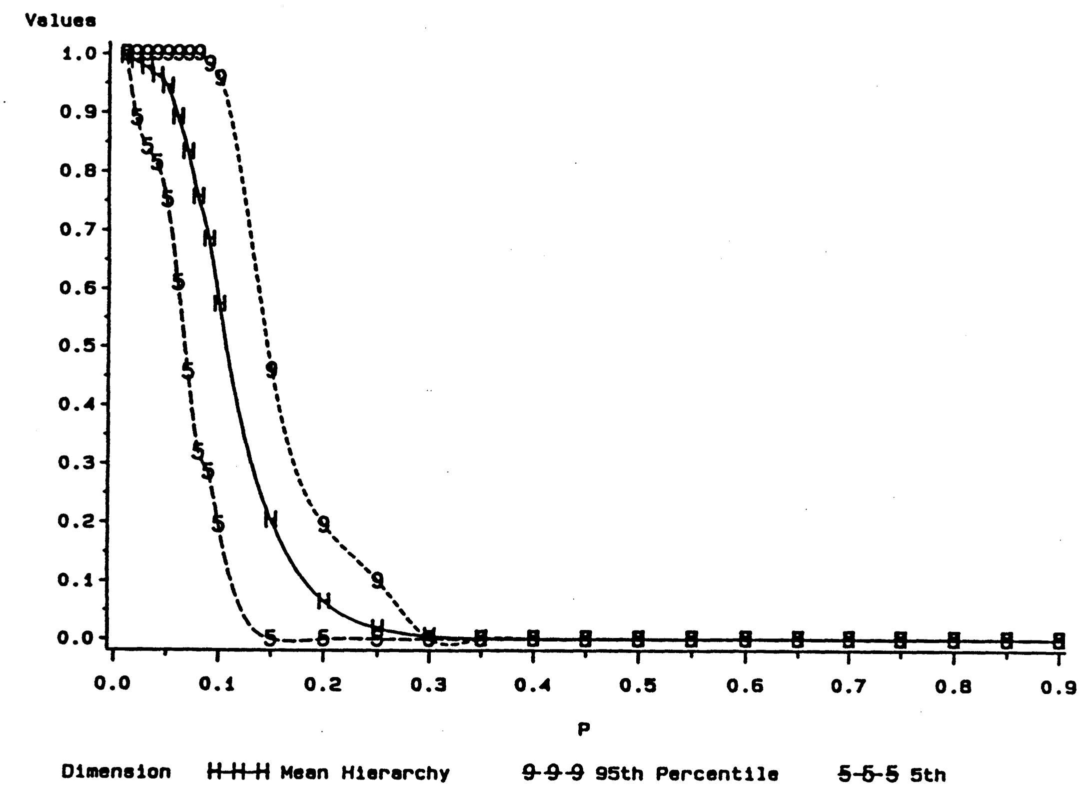


FIG. 5.A7. N=20.

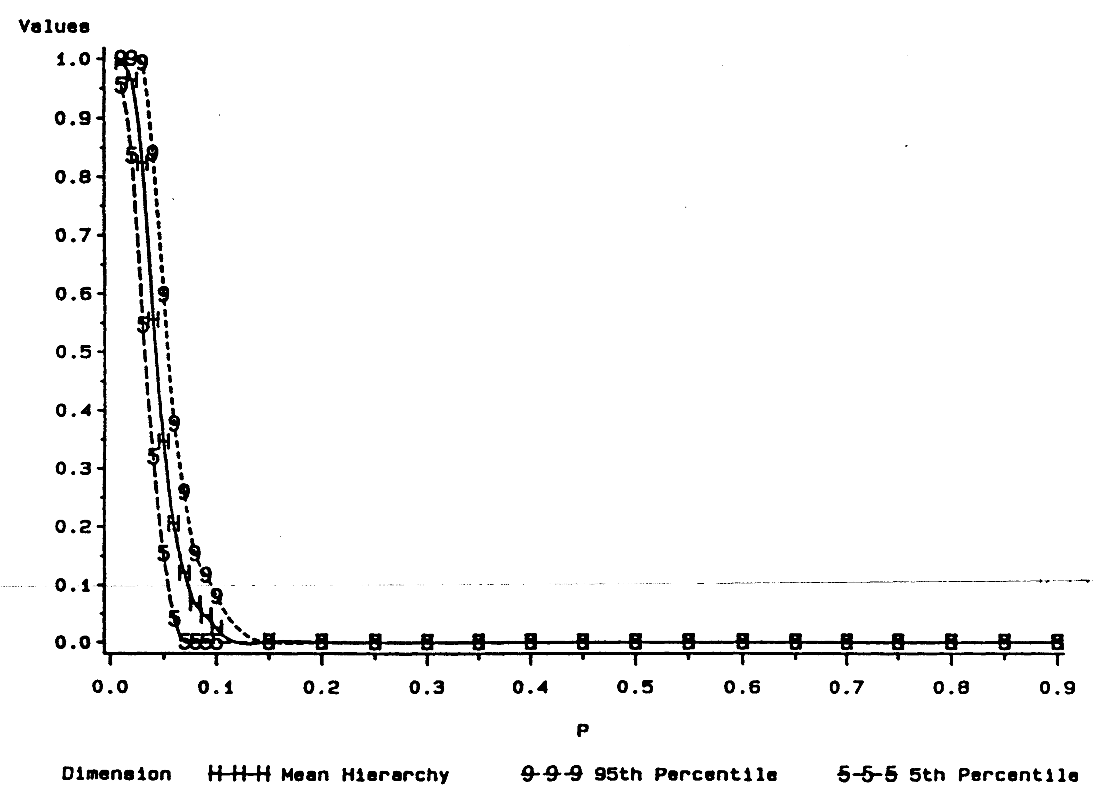


FIG. 5.A8. N=50.

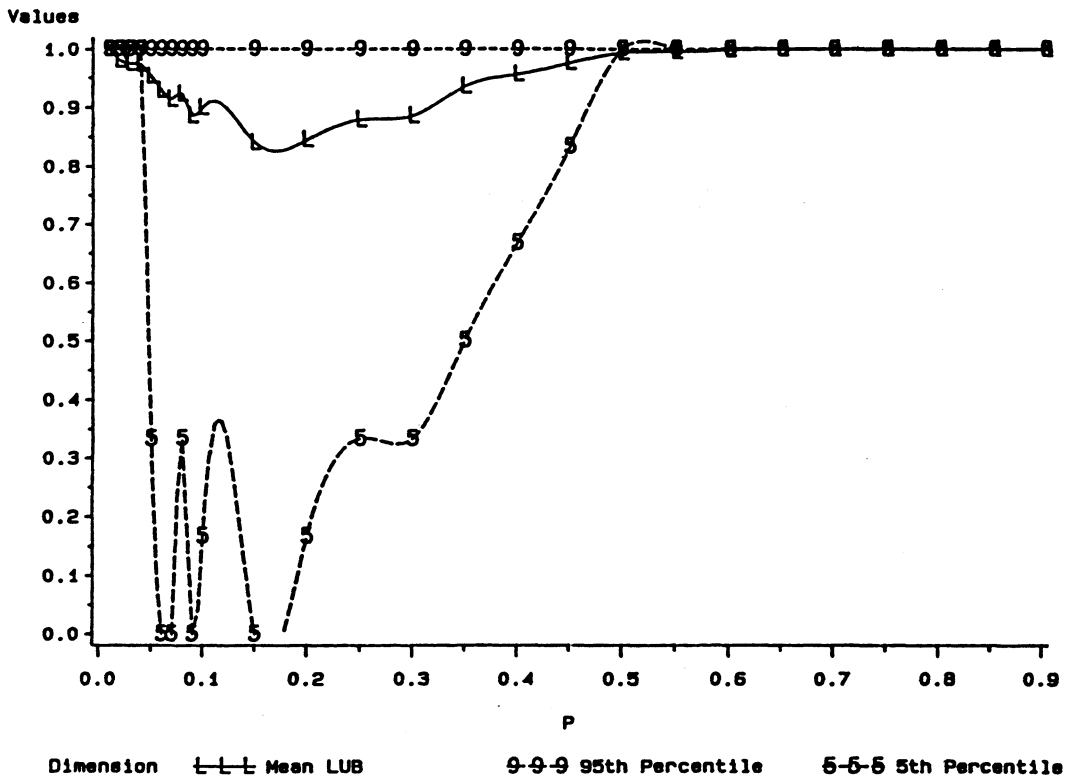


FIG. 5.A9. N=5.

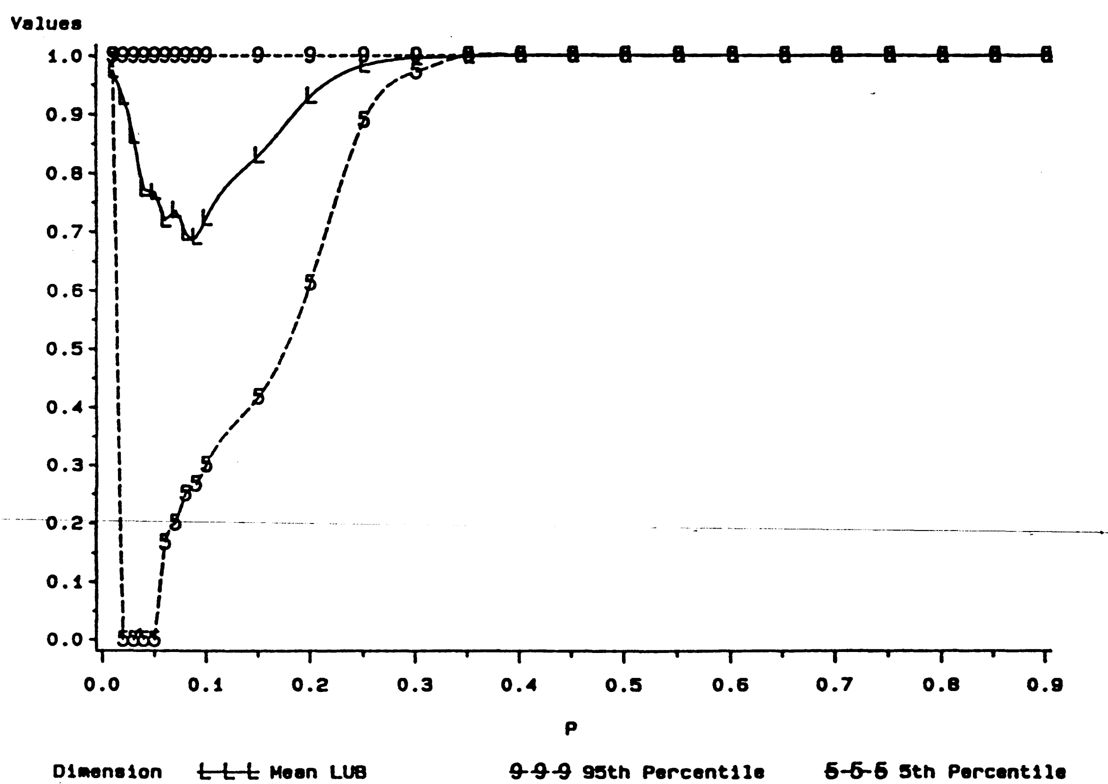


FIG. 5.A10. N=10.

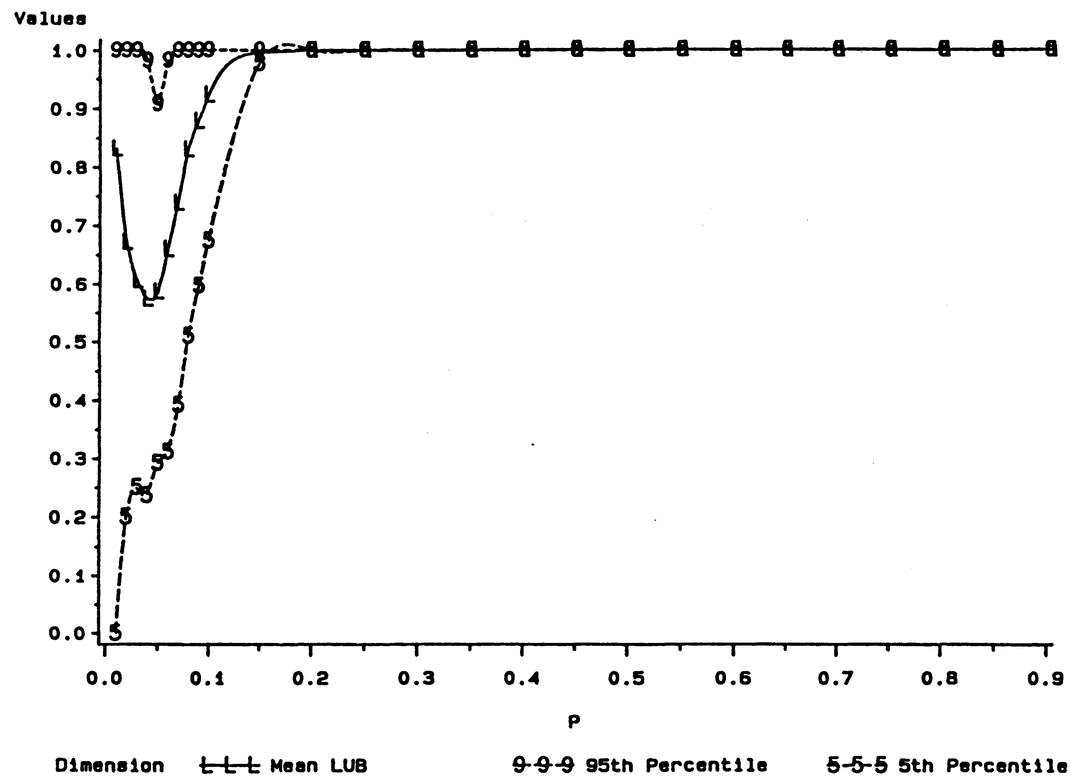


FIG. 5.A11. N=20.

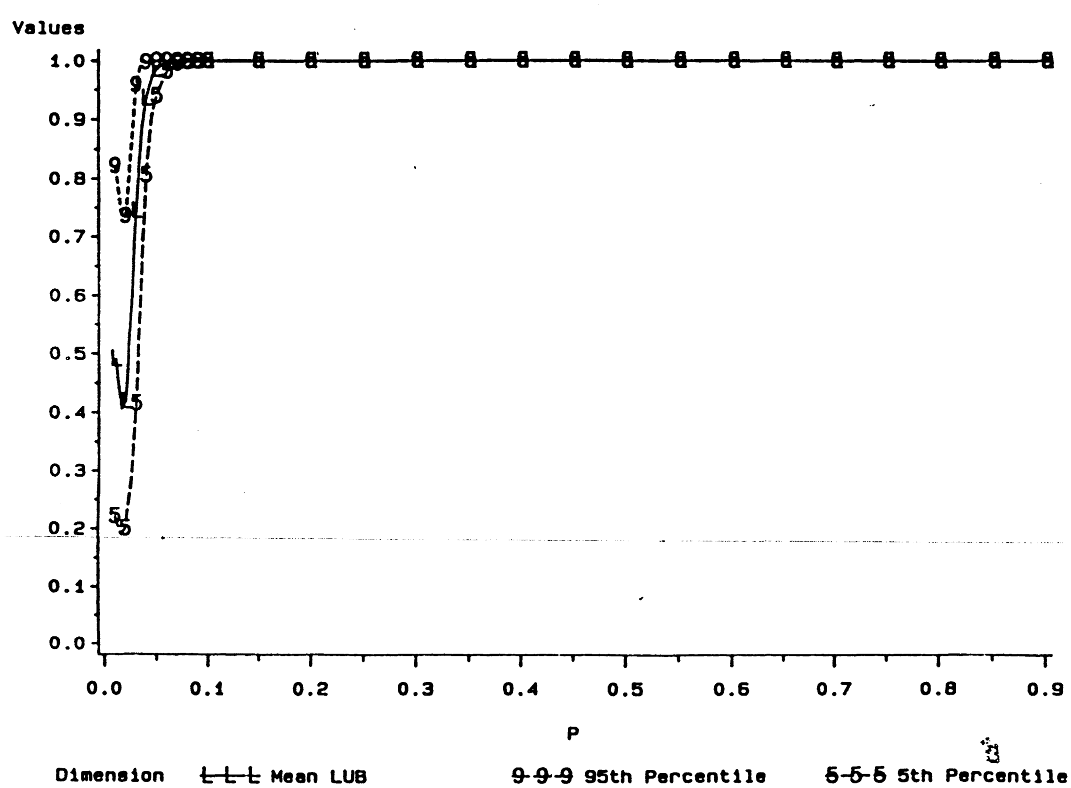


FIG. 5.A12. N=50.

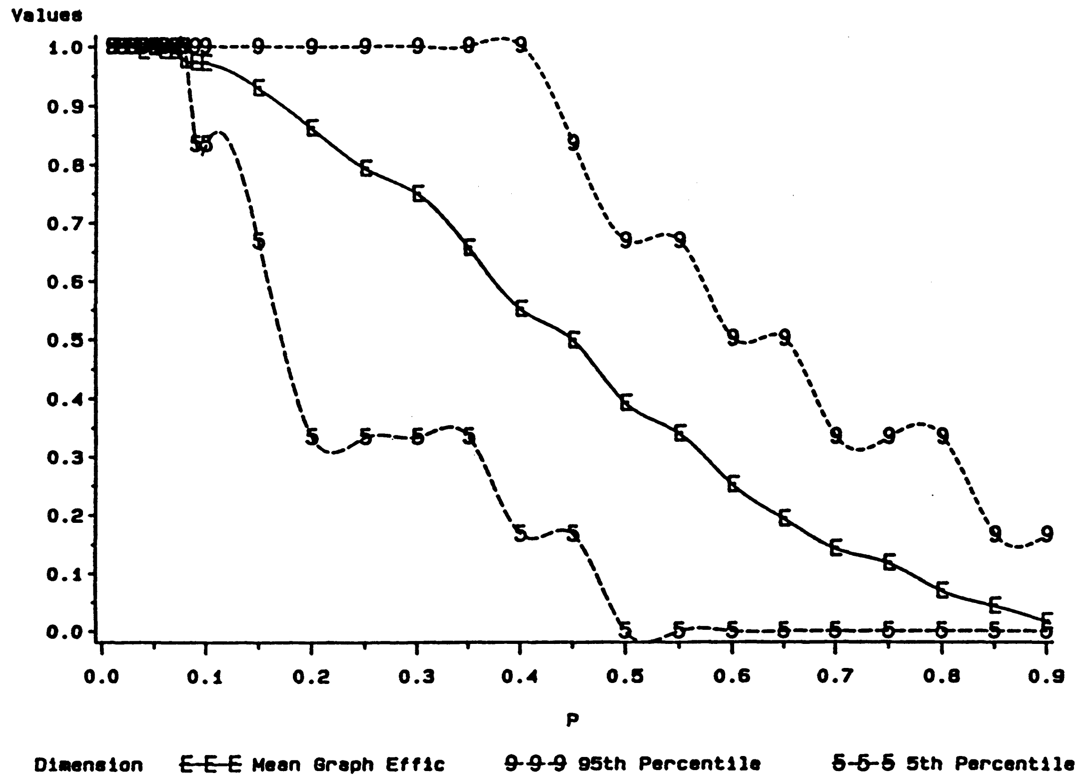


FIG. 5.A13. N=5.

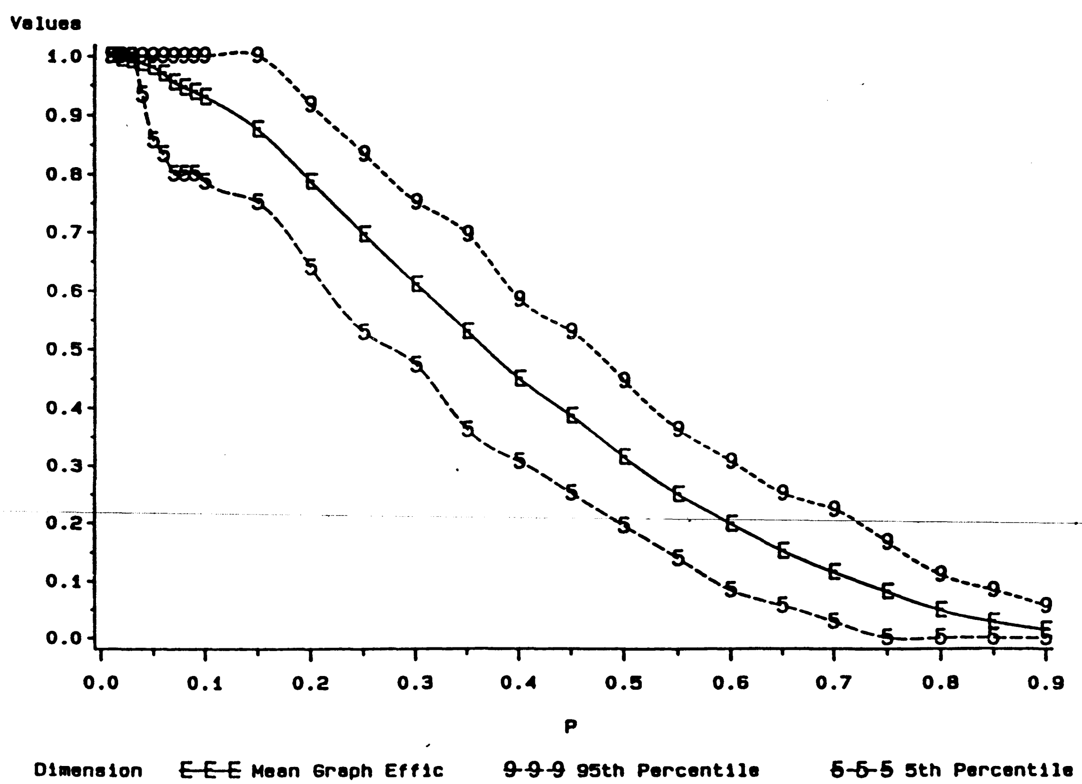


FIG. 5.A14. N=10.

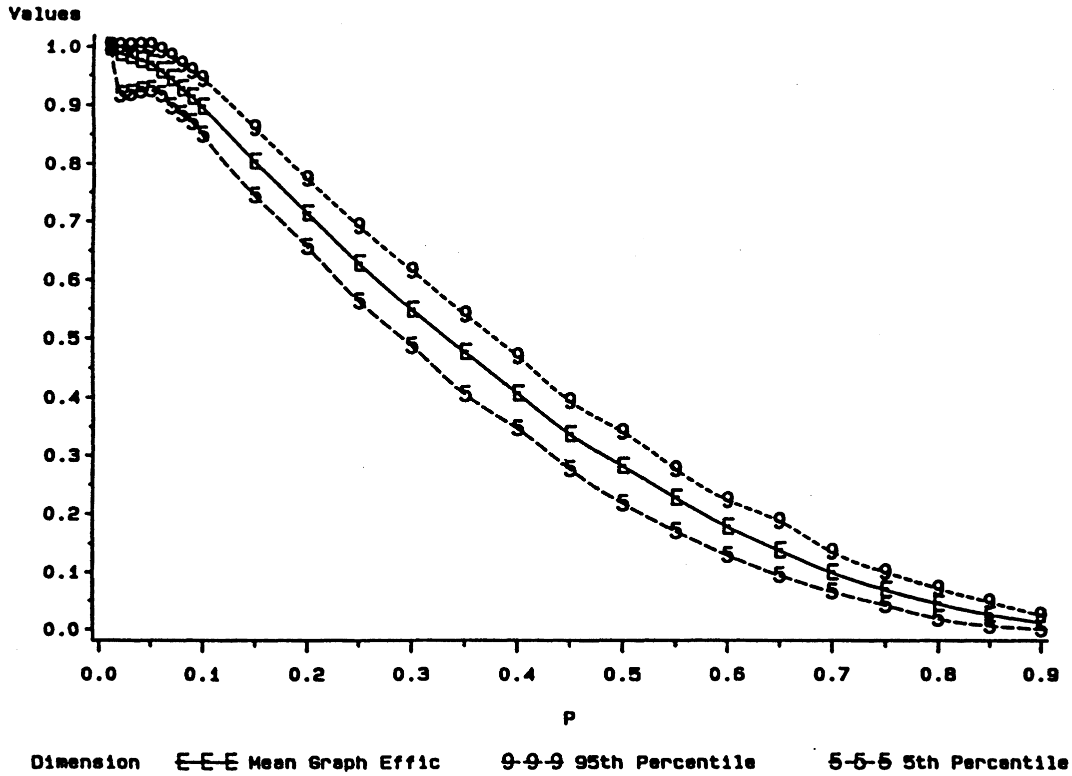


FIG. 5.A15. N=20.

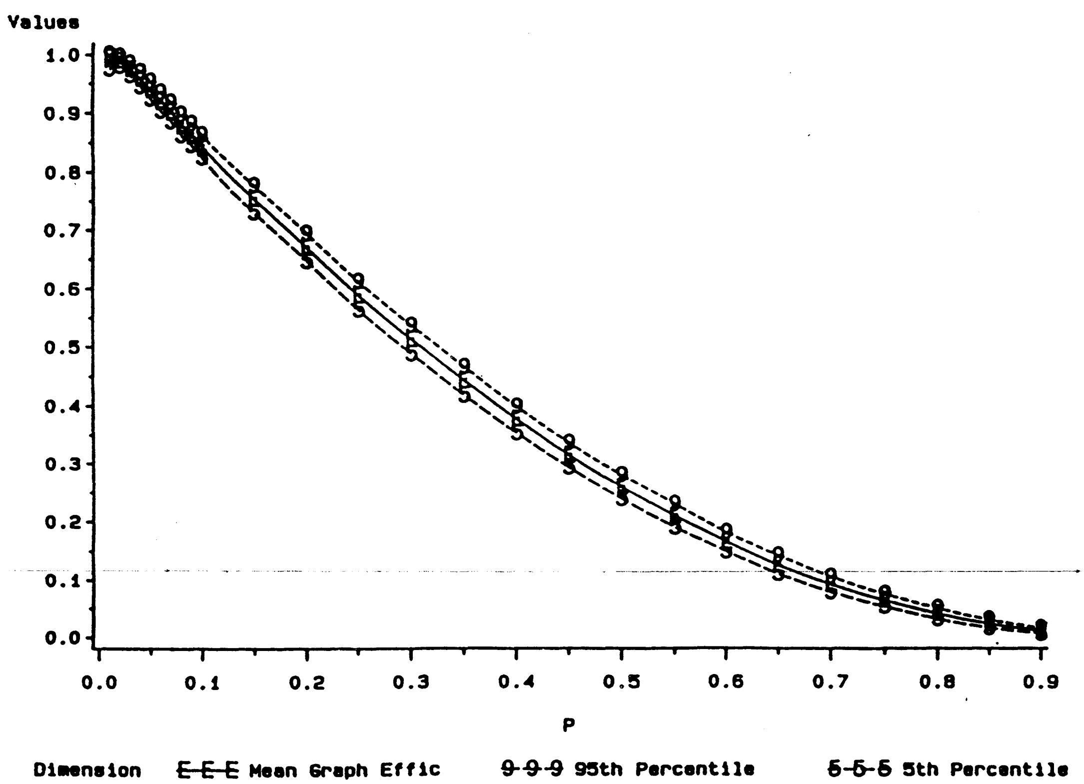


FIG. 5.A16. N=50.

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# Computational Organization Theory

*edited by*
**Kathleen M. Carley**
**Michael J. Prietula**
*Carnegie Mellon University*

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