Reasoning About Networks Over Time: Types of Dynamical Analysis

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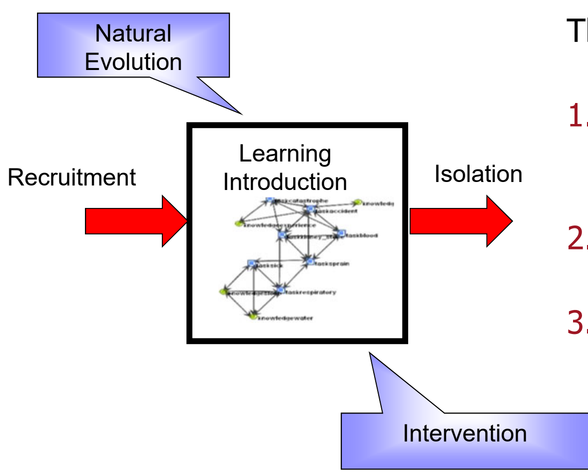
Change in Networks



Three Ways to Study Dynamics

1. Comparison over time
   – Look at real data
2. Immediate Impact
   – Comparative statics
3. Near Term Impact
   – Utilizes simulation - DyNet

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Basic Issue

- Over time the set of nodes change
- What should you do?
  - Compare just nodes present in all time periods
    - For core group – how has it changed
  - Create a master network of all nodes
    - How has the flux altered the groups
  - Use whatever nodes are available
    - What are the natural dynamics
- No single right answer
  - It depends on what you want to know
  - It depends on how your underlying network changes with time
  - Often try two different approaches and see how much they differ

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Types of Over-Time Change in Networks

- Stability
  - Relationships remain the same over time.
- Evolution
  - Interaction among agents cause the relationships to change over time.
- Shock
  - Change is exogenous to the social group.
- Mutation
  - A shock stimulates evolutionary behavior.

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Models Used to Study Change

- Use different models for different types of Change
  - Stability: LPM , ERGM, repeated measures
  - Evolution: SIENA, multi-agent simulation, or both
  - Shock: Change detection in real-world applications
    Multi-agent simulation for experimentation
  - Mutation: Change detection coupled with SIENA for real world applications
    Multi-agent simulation for experimentation

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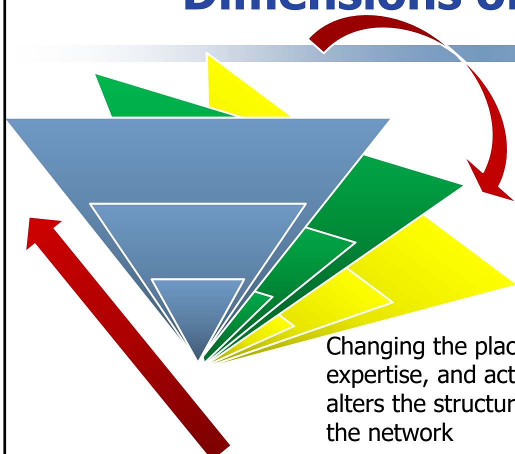
Dimensions of Relevance



Changing the place, expertise, and activity alters the structure of the network

Increasing the time, spatial, and group span reduces the information loss but increases complexity

Aggregators – determine what observed
- Distance - Space
- Time
- Group

- Impacts analysis
  - Degree of resolution
  - Information Loss
- Impacts hierarchical or level effects
  - Slower change
  - Wider consequences
  - Inheritance

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Social Networks are Continuously Emerging Structures

- Networks emerge from intersecting trails
  - Constrained and enabled
- Networks reinforce some trails
  - Secondary emphasis to some constraints
- Slices across trails are the "measured" or "observed" social network
- The level of aggregation determines the "width" of the slice
  - The greater the width – the higher the density of connections

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Aspects of Trails of Interest

- PLACE – Physical
  - Who was where when
  - doing what (how (to/with whom (why)))
- EXPERTISE – Cognitive
  - Who was providing what information when
  - how (to whom (from where (why)))
- ACTIVITY – Occupation
  - Who was doing what when
  - how (with whom (where (why)))

Trails Provide Meta-Network Information

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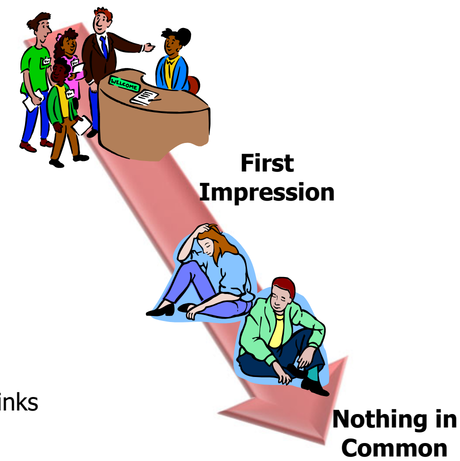
Social Dynamics due to Learning

- Implicit link
  - seen together
  - common sources
  - seniority
- Explicit link
  - information exchange
  - learned from each other
  - mentoring



- When meeting a new person
  - Infer expertise based on implicit links
  - Baseline for trust
  - Social shakeout occurs as you move from implicit to explicit links

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Topic 1: What if Key Actors are Removed?

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Critical Personnel

- Individual whose absence will dramatically decrease performance of organization
  - Only person who can do a task
  - Only person with certain organizationally critical knowledge
  - Person who keeps others in line, supported, feeling good about the organization
  - Person who is the only access point to certain organizationally critical knowledge
  - Only person who knows key people
  - Person who knows almost everything
- Examples
  - Lead scientist
  - Life time administrative assistant
  - One person lab/technician
  - Lone visionary

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Critical Personnel

- Key players, network elite
- Those with power
- Those who, were they to leave, would reduce the organizations performance, adaptability, competence …
- Direct identifiers
  - The centralities: e.g., degree
  - The exclusivities: e.g., task
  - The integrators: e.g., simmelian ties
  - The loads: e.g., workload and cognitive demand
- Indirect
  - Those who have access to, can influence, those who are critical

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What Happens When Critical Personnel are Removed

- Decrease redundancy
- Decrease or remove intellectual property
- Alter performance
- Alter adaptability
- Alter information diffusion

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How to be less vulnerable and more adaptive

- Increase redundancy
  - Decrease number of tasks – outsource
  - Decrease number of skills/resources/knowledge – do simpler tasks, employ skill reduction technologies
  - Increase number of personnel
  - More highly train personnel (each knows more)
  - Increase workload
  - Redistribute workload – retask individuals
  - Redistribute resources – retrain individuals
  - Redistribute employees – reassignment

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Networks Heal Themselves

- Networks, particularly cellular networks, can withstand high levels of turnover
- Agents the are in structurally "equivalent positions" are replaceable by others that are "equivalent"
  - Connected to same others
- Agents in specialized positions, e.g., those with high cognitive load, are harder to replace
- Newcomers typically enter as neither structurally equivalent with a key actor nor high in cognitive load
  - Low transactive memory
  - Few pre-existing ties
  - "start off on simple tasks"

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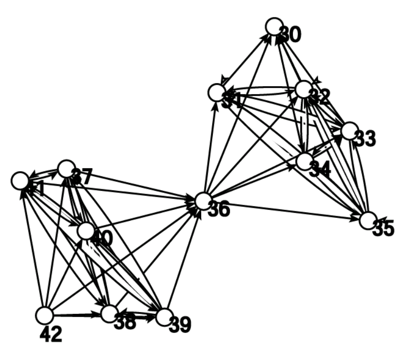
Network Evolution

- Causes
  - Learning
  - Physical movement
  - New resources
  - Attrition
  - Removal
- Interaction Logics
  - Homophily
  - Information seeking
  - Co-work
  - Co-location



1. Original Cell Configuration

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Network Evolution

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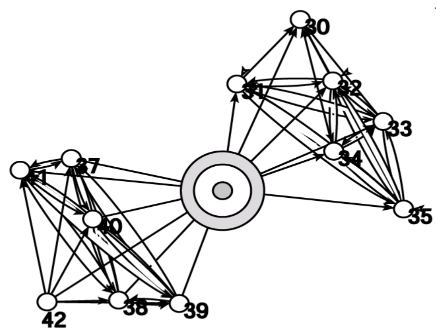

2. Node 36 Targeted for Isolation

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Network Evolution

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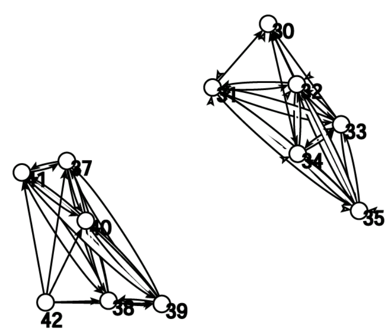

3. Two Cells are Disconnected

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Network Evolution

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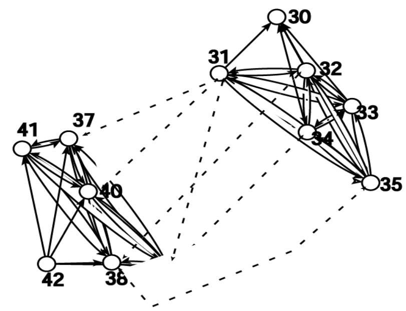

4. Agents use referential data to attempt to reconnect cells

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Network Evolution

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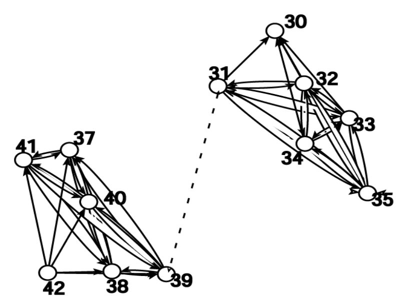

5. One of the connections succeeds
Others are refused

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Network Evolution

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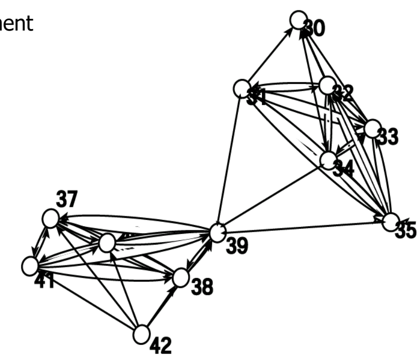

6. New Cell Leader Has Emerged

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Immediate Impact - Prediction

- *What if ? Remove top 5 emergent leaders*
- Change in performance
  - Anticipated drop – 4% percentage difference
- Change in information diffusion
  - Anticipated increase – 67% percentage difference
- New emergent leaders
  1. 0.0174 said_mortazavi
  2. 0.0137 kamal_kharazi
  3. 0.0127 reza_asefi
  4. 0.0120 morteza_sarmadi
  5. 0.0100 hashemi_shahroudi
- Value of "lowest" old emergent leader was .0246

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Topic 2: Over Time Networks - Setup

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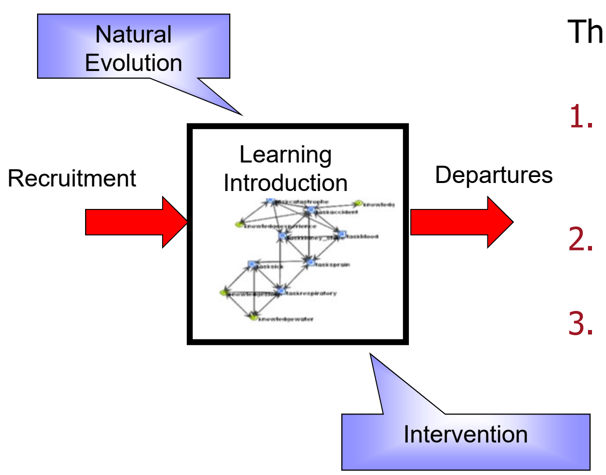
Changes in Networks over Time



Three Ways to Study Dynamics

1. Comparison over past times
   – Look at real data
2. Immediate Impact
   – Comparative statics
3. Near Term Impact
   – Utilizes simulation - DyNet

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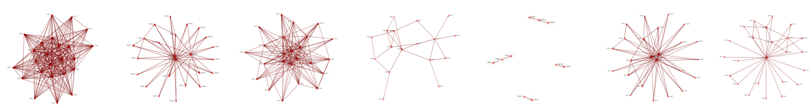
Longitudinal (Over Time) Networks

- Consider watching communications on a network, such as email. Mark a link between agents if communicated.



- Has this organization changed significantly?
- Has it evolved?
- Have people changed their position in the network?

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One Issue: the node set

- Over time the set of nodes change
- What should you do?
  - Compare just nodes present in all time periods
    - For core group – how has it changed
  - Create a master network of all nodes
    - How has the flux altered the groups
  - Use whatever nodes are available
    - What are the natural dynamics
  - Note – choice changes many measures that are scaled by size
- No single right answer
  - Right answer depends on what you want to know
  - Often try two different approaches and see how much they differ

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Taxonomy of Change in Network Data

- Stability: Relationships remain the same over time.
  - But will still have significant "random" variations with time
- Evolution: Interaction among agents cause the relationships to change over time.
  - Normal state of affairs with humans beings as agents
  - Still has "random" variations as well
- Shock: Change is exogenous to the social group.
  - This is crucial for many real world applications
- Mutation: A shock stimulates evolutionary behavior.
  - This is longer term response of organization to changing environment

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Models Used to Classify "Change"

- Stability: LPM , ERGM, repeated measures
  - LPM is Link Probability Model
  - ERGMs are Exponential Random Graph Models
- Evolution: SIENA, multi-agent simulation (CONSTRUCT), or both
- Shock: Change detection in real-world applications
  Multi-agent simulation for experimentation
- Mutation: Change detection coupled with SIENA for real world applications
  Multi-agent simulation for experimentation

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Dynamic Analysis Techniques

- Visualization
- Comparative Statics – Immediate Impact
- Longitudinal Networks and Change
  - Stability, Evolution, Shock, Mutation
- QAP (Quadratic Assignment Procedure) and MRQAP (Multiple Regression QAP), Longitudinal QAP
- Statistical Models of Networks
  - Link Probability Model (LPM) for Stability
  - Actor-Oriented Models for Evolution
  - Multi-Agent Simulation for Evolution, Shock, and Mutation
- Exponential Random Graph Models
- SIENA
- Statistical Process Control
- Network Change Detection
- Fourier Analysis
- Simulation (Agent-Based Dynamic Network)

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Communications as a Proxy

- "Ideal approach" – directly sample network each time period
  - E.g., have every member of society fill out survey every time period
  - Limited to very small societies and really motivated subjects
- Or, tracking changes over time using communications data
  - Communication is "proxy" for a network tie
  - Tracking large amounts of communication data gives approximate picture of the underlying social network structure
  - Can use it to find Key Entities and other Network measures
- Communication log data available from many sources
  - Cell Phone Service Providers – call logs, txt msg logs
  - E-mail Data logs – available within organizations
  - Software: Twitter, Facebook, FourSquare, etc.
  - Hardware: building sensors, cell phone sensors, RFID Tags, GPS, etc.

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Communications Log Data

- Data on who you talk to over monitored means, but NOT what you say (decreased privacy concerns relative to full text monitoring)
- Researchers often only have access to logs from 1 or 2 communications channels – not all possible channels
  - Missing data is substantial
- Communication event is taken as a proxy for a link
  - But this may not always be the case; e.g., calling a wrong #

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Is Com Log Data a good Proxy ?

- Example: 2011-2013 NetSense Data Set from Notre Dame
  - Aaron Striegel, Shu Liu, Lei Meng, Christian Poellabauer, David Hachen, Omar Lizardo, "Lessons Learned from the NetSense Smartphone Study," Proceedings of HotPlanet'13, August 16, 2013, Hong Kong, China.
- They recruited 180+ incoming freshmen/freshwomen in 4 dorms to join study
  - Students received free cell phone (including phone plan)
  - Students had to agree to use provided Android cell phone as their primary cell phone
  - Students agreed to having calls and txt msgs logged
  - Students agreed to filling out monthly surveys
- Data collected from study for 4 academic semesters
  - Data from Summer survey too unreliable to use because many students were away from campus for summer

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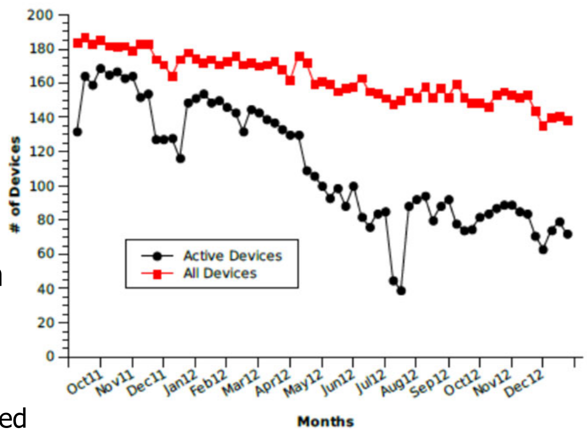
NetSense Details

- NetSense study surveyed all participants monthly + an extra long survey at end of each semester
  - Survey return rate nearly 100%
  - This work focused on the long survey at end each sem.
  - Long Survey Question asked top 10 people you interact with
- NetSense population changes over time
  - Students either quit or violate terms of study and are removed



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Methodology

- Question to be studied:
  - Accuracy of phone logs relative to survey for predicting network
- Survey
  - Asked students to list top 10 people they interact with regularly
  - Students didn't have to fill in all 10 slots
  - May of those listed were people outside of study (e.g., parents)
  - Keeping only those in study gave list of 0-10 others in the study that the surveyed individual considered strong interaction targets
- Cell Phone Data
  - Looked at # txt msgs, # txt chars, # phone calls, # secs on calls
  - Ranked in-study interactors based on these metrics
- Predictor Quality
  - Probability individual listed as one of N in-study individuals in survey is in the top N based on cell phone data

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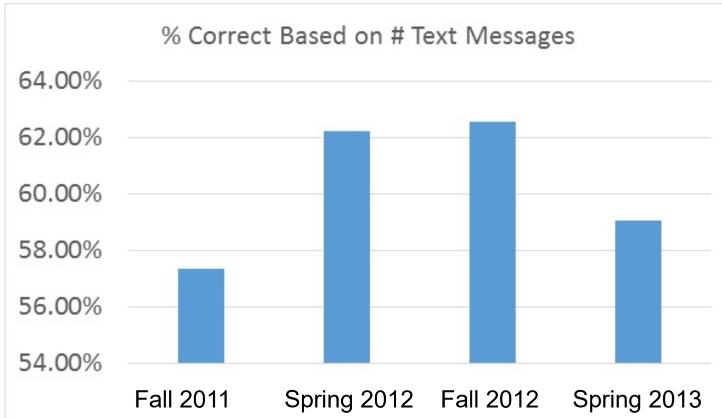
Text Messages

- This generation lives on text messages
  - Overall, # txt msgs accurage about 60% of the time



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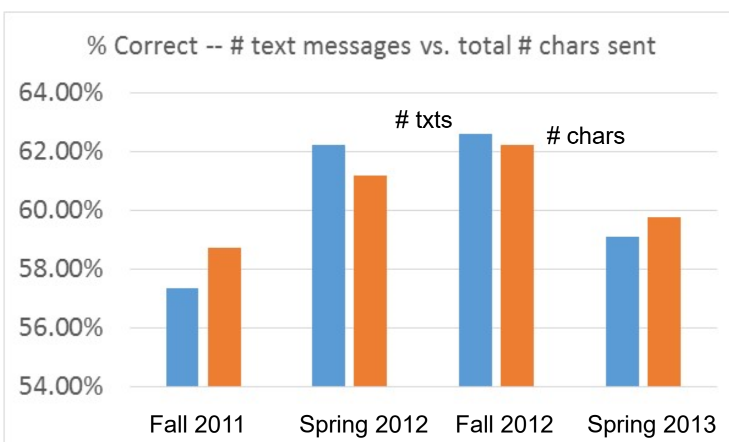
Chars in Text Messages

- Length of text message does not seem significant
  - Using # chars instead of # txt msgs is actually slightly worse



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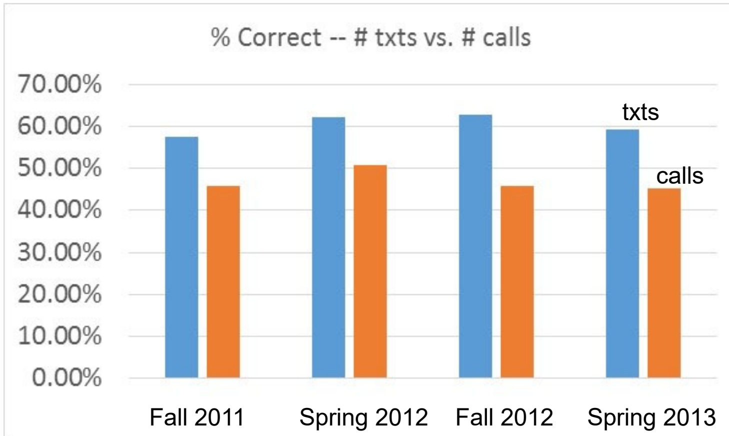
Phone Calls

- How about using # phone calls instead of # txt msgs
  - Not good at all – only about 45% on average for phone calls



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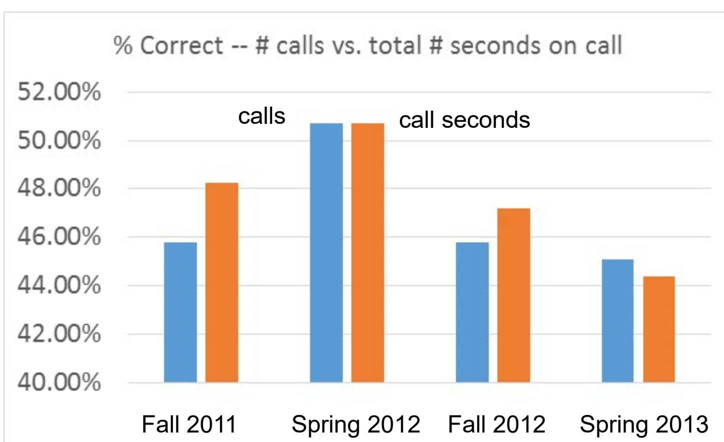
Call Seconds vs. # Calls

- How about using # call seconds instead of # txt msgs ?
  - A little better – about 48% on average for calls seconds



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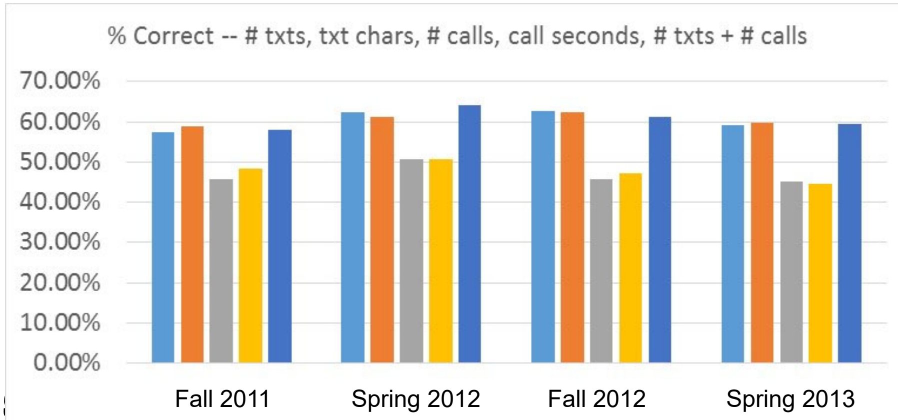
Composite Measures

- Comparing all to (#txts + #calls) as measure ?
  - This combination achieves highest score, above 60% accurate



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Conclusions: Com Logs can be OK Proxy for Network Ties

- # txt msg is good proxy for interaction propensity for this cohort
- Combinations of comm data metrics can slightly increase accuracy, but only a little
- Accuracy level of about 60% indicates that many interactions are mediated by other communications channels (e.g., face-to-face).
- Results of this analysis may vary widely for different communities – in 2011, freshmen/freshwomen are highly attached to txt msgs for communication
- Note, self-reporting errors may influence these results – e.g., participants took final survey less seriously

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Critical Issue: Slicing and Dicing

- Approach 1: Cumulative network
  - Each time period is all prior links plus new
  - Good for data where links don't go away – e.g., citation networks
- Approach 2: Divide based on external shock
  - Number of time windows depends on external events e.g., before and after a referendum
  - Good for data where there is a major known change
- Approach 3: Divide into uniform periods
  - Number of time windows depends on collection and time slice
  - Good for large data and for doing periodicity studies
- Approach 4: Streaming
  - Only show most recent data using some moving average
  - Good when data too large to be stored – least developed

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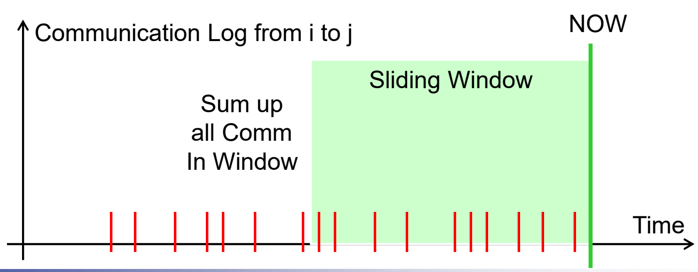
Sliding Window for Over-Time Links

- Estimator for Link Weight (a.k.a. Link Cost)
  - Add up # of Communication Events between x & y in window
  - Take reciprocal. If # is 0, there is no Link between that pair
  - Then move window forward by a time step and repeat
  - Alternatives possible:
    - Incorporate duration of communication
    - Weight different communications channels differently



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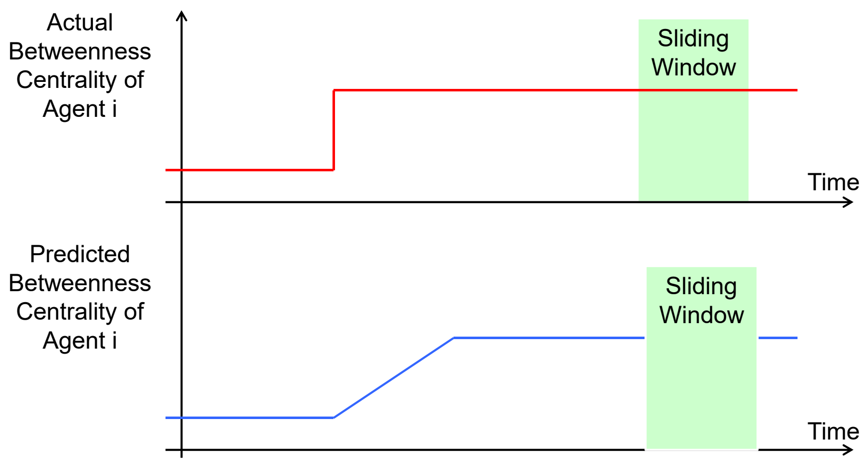
Smoothing Effect of Sliding Window



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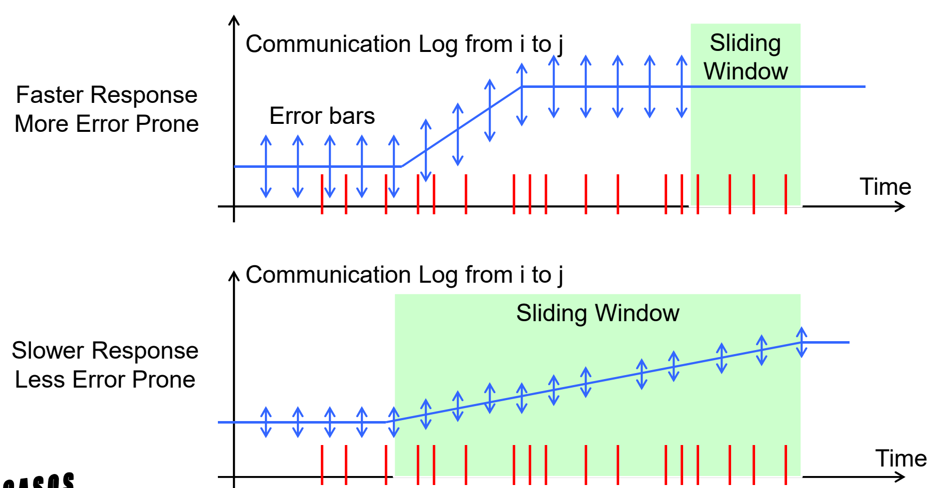
Adjusting Window Size



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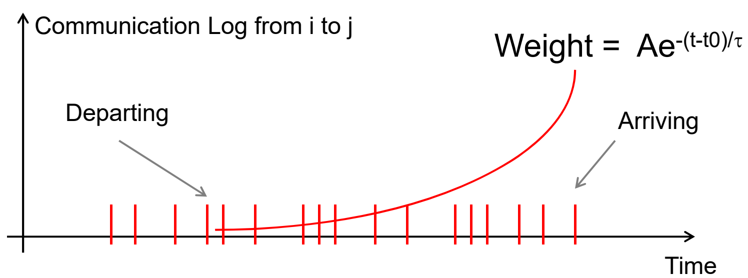
Mathematically Better Window

- Improved tradeoff between smoothing and averaging
  - Mathematically, Exponentially Weighted Moving Average (EWMA)
    - Considers all past known events in estimating current network
    - Old events receive smaller and smaller weighting
    - New events receive highest weighting
    - Exponential time constant – τ – sets how quickly past attenuates vs. how much averaging reduces variance of network



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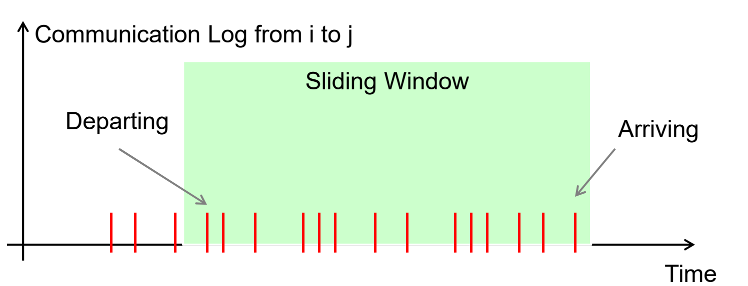
Incremental Sliding Window

- Sliding Window is Synergistic with Incremental Analysis
  - As window moves forward in time
    - New events "arrive" and must be processed
    - Old events "fall out" of trailing edge of window and must be processed
    - BUT – all of the data in middle of window remains unchanged
    - Incremental algorithms fast because only small part of data changes



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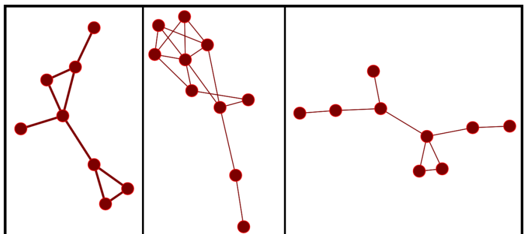
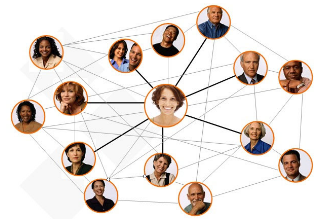
Dynamic Metrics on Over-Time Data

- *Identifying central nodes in a network*



T_1 T_2 T_3



Dynamically Changing Network Structure!!!

Focus on calculating measures for each network
Then evaluate how measures change over time

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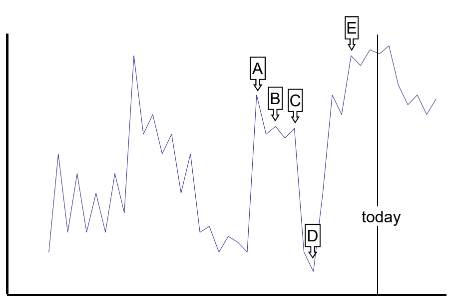
Changes in Network Data Measures

- Various measures of a network are calculated for a window of network data at a multiple points in time
- <u>Change detection</u>: quickly determine *that* a change occurs.
- <u>Change point identification</u>: *when* did the change occur.



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Change Detection

- Goal: Rapidly detect that a change has occurred
- Detect *shocks*, not evolutionary changes
  - Evolutionary change: change due to interaction among actors in a network
    - Example: change of interaction patterns over time among new students as they get to know each other
  - Shock: change reason is exogenous to the network
    - Example: change of interaction patterns among students after they graduate
  - Another way to say it: detect "fast" change not "slow" change
- Another goal is to identify *change point*
  - Likely time when change occurred
  - Limits the scope of explanation for network change

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Theory of Change and Network Evolution – is it "Change"

- Need a theory for how links form over time – Null Hypothesis
- Random – assume Network links appear at random
- Heiderian balance
- Blau exchange
- Socio-Cognitive needs
  - Homophilly
  - Expertise
  - work
- The Rich get Richer
  - Popularity
    - Most likely link is to nodes that others link to
  - Preferential attachment
    - Variation on the theme
  - Limits to growth/interest
    - Link to those not over-committed

Common Biases

- Interaction Logics/Biases
  - Homophilly
    - Relative similarity
  - Relative expertise
  - Need to work
  - Need to coordinate
  - Activity
  - Node intelligence
  - Preferential attachment
  - Distance decay
- Often referred to as generative Grammars

Random

- Network ties are random
- Each time period just generate a random network of a particular size and density
  - Size and density may grow or shrink via other models
- The "naïve" baseline

Classic Random Graph Models

- In the **G(n,p)** random graph model:
  1. There are ***n*** nodes.
  2. There is an edge between any two nodes with probability ***p***.

Proposed by Erdös and Renyi in 1960s.

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Properties of Online G(n,p)

- E[degree of first node] = $1 + 1/2 + 1/3 + 1/4 + \ldots 1/n$ = $\Theta(\log n)$

- E[max degree] = $\Theta(\log n)$

- X_k = Proportion of nodes with degree k
  $E[X_k] = \Theta(½^k)$

This does NOT generate a POWER LAW

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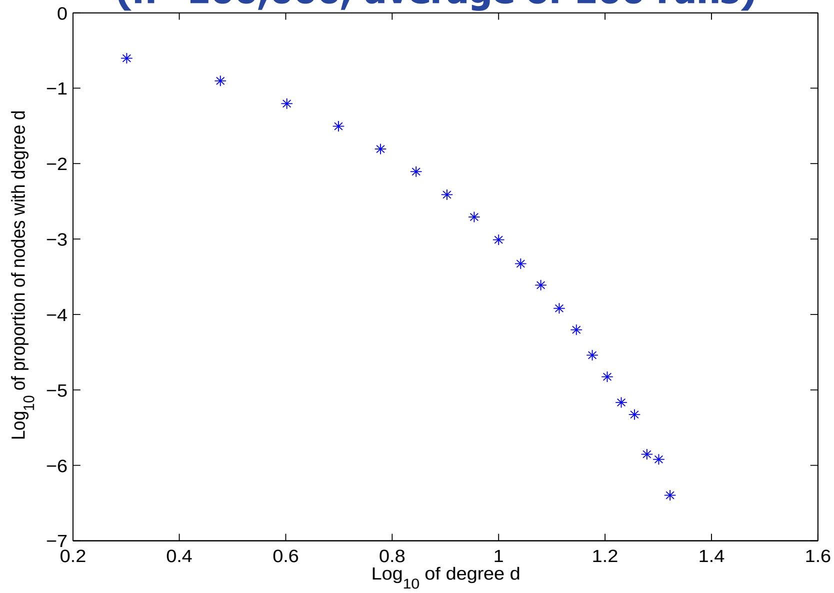
Online G(n,p)

(n=100,000, average of 100 runs)



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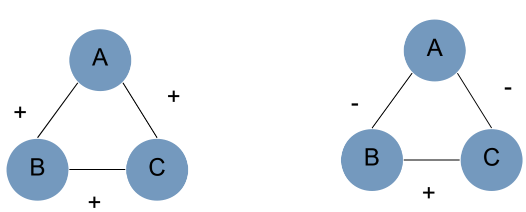
Heiderian Balance

- Instead of 0/1 Links, let us allow -1 / 0 / 1 links
- Actors are only comfortable in balanced relations
- Balance is achieved when there are three positive links or two negatives with one positive.
- Two positive links and one negative creates imbalance.



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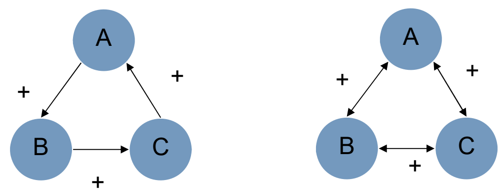
Blau Exchange

- Exchange strengthen ties
- Tendency to reciprocity
- Reciprocity is strongest when in triadic relations



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Theory Based Inference: Meta Network

- Homophilly
  - Knowledge
  - Resources
  - Attributes
  - Etc
- Two mode networks needed:
  - Such as People by expertise or People by resources
- Operationalized as
  - Similarity
  - Relative similarity
  - Similarity on shared and non shared characteristics
  - Relative similarity on shared and non shared characteristics

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Interaction Style: Need for Communicative Ease - Homophily

- Relative similarity = how much i shares with j divided by how much i shares with all others
- AKik is knowledge network
  - Knowledge network is agent by knowledge ("facts")
- Expected interaction based on relative similarity

I = max number of agents
K = max number of ideas, facts, pieces of knowledge

$$RSij = \frac{\sum_{k=0}^{K} (AKik * AKjk)}{\sum_{j=0}^{I} \sum_{k=0}^{K} (AKik * AKjk)}$$

$$Cutoff = \sum_{i=0}^{I} RSij \; / \; (I * (I - 1))$$

If RSij $\geq$ Cutoff the Expected interaction = 1
else 0

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Relative Similarity - Why

- Similarity: individuals tend to interact with those whom they deem to be more similar to themselves
  - Comfort
  - Ease of interaction
  - Ease of access
  - Common language
  - More effective for getting information
  - Shared expectations about reciprocity
- Relative: individuals judge similarity relative to others
  - There is a comparison group
  - There is a generalized other

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Interaction Style: Need to Know Relative Expertise

- Relative expertise = how much i thinks j knows that i does not know divided by how much i thinks all others know that i does not know
- AKik is knowledge network
- Expected interaction based on relative expertise

I = max number of people
K = max number of ideas

If $AK_{ik} = 0$ then $X_{jk} = AK_{jk}$
Else $X_{jk} = 0$

$$RE_{ij} = \frac{\sum_{k=0}^{K} (X_{jk})}{\sum_{j=0}^{I} \sum_{k=0}^{K} (X_{jk})}$$

$$\text{Cutoff} = \sum_{j=0}^{I} RE_{ij} \; / \; (I * (I - 1))$$

If $RE_i \geq$ Cutoff the Expected interaction = 1
else 0

Relative Expertise - Why

- Expertise: individuals tend to interact with those whom they believe to have information that they need
  - Information gathering
  - Desire to achieve
  - Desire for increase in power
  - Information as power
- Relative: individuals judge expertise relative to others
  - There is a comparison group
  - There is a generalized other

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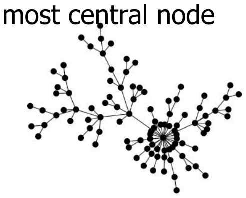
The Rich Get Richer Centrality Increases Models

- Popularity – rich get richer
  - As size goes up new nodes link to most central node
  - On average
- Preferential Attachment (Yule or Matthew effect)
  - New nodes are connected to old according to the number of others already connected
  - Can generate power laws
- Limits to Growth
  - As size goes up new nodes are added to the most central node that has not hit its limit
  - On average



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Preferential Attachment

- In the Preferential Attachment model, each new node connects to the existing nodes with a probability proportional to their degree.

(1) *Growth*: Starting with a small number (m_0) of nodes, at every timestep we add a new node with $m(\leq m_0)$ edges that link the new node to m different nodes already present in the system.

(2) *Preferential attachment*: When choosing the nodes to which the new node connects, we assume that the probability Π that a new node will be connected to node i depends on the degree k_i of node i, such that

$$\Pi(k_i) = \frac{k_i}{\sum_j k_j}$$

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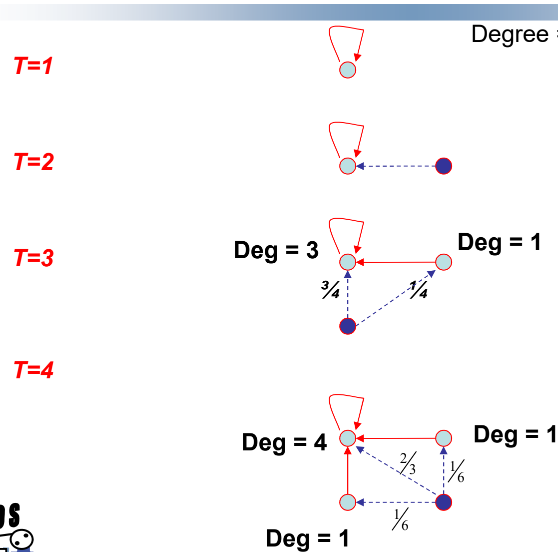
Preferential Attachment

Degree = in-degree + out-degree



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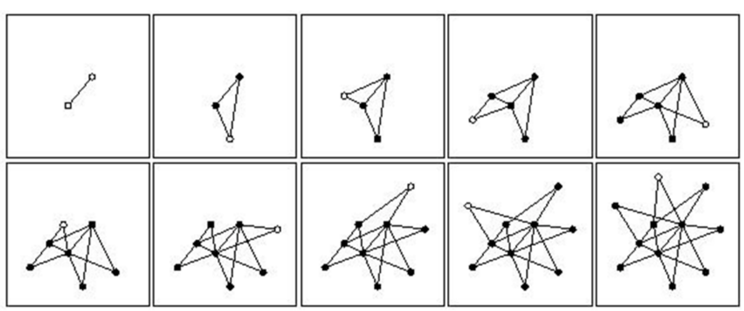
Preferential Attachment

- Linear preferential attachment processes in which the number of nodes increases are known to produce a distribution of node centralities following the so-called Yule distribution.
- The fraction of nodes having k links in the limit $P(k) = \frac{\mathrm{B}(k+a,\gamma)}{\mathrm{B}(k_0+a,\gamma-1)},$



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Measuring preferential attachment

- Is it the case that the rich get richer?
- Look at the network for an interval [t,t+dt]
- For node i, present at time t, we compute

$$D_i = \frac{dk_i}{dk}$$

  - dk_i = increase in the degree
  - dk = number of edges added
- Fraction of edges added to nodes of degree k

$$f(k) = \sum_{i:k_i=k} D_i$$

- Cumulative: fraction of edges added to nodes of degree at most k

$$F(k) = \sum_{j=1}^{k} f(j)$$

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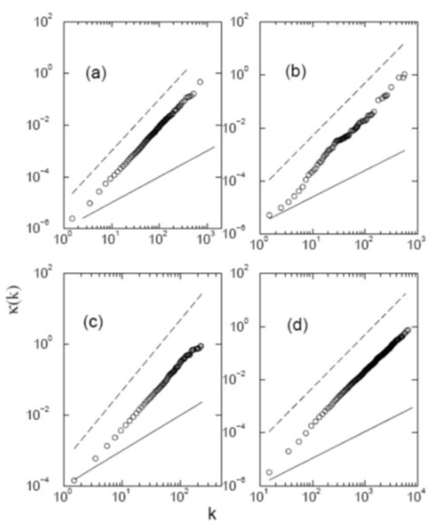
Measuring preferential attachment

- plot F(k) as a function of k



(a) citation network
(b) Internet
(c) scientific collaboration network
(d) actor collaboration network

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Preferential Attachment

E[degree of 1st node] = $\sqrt{n}$

Preferential Attachment gives a **power-law** degree distribution. [**Mitzenmacher, Cooper & Frieze 03, KRRSTU00**]

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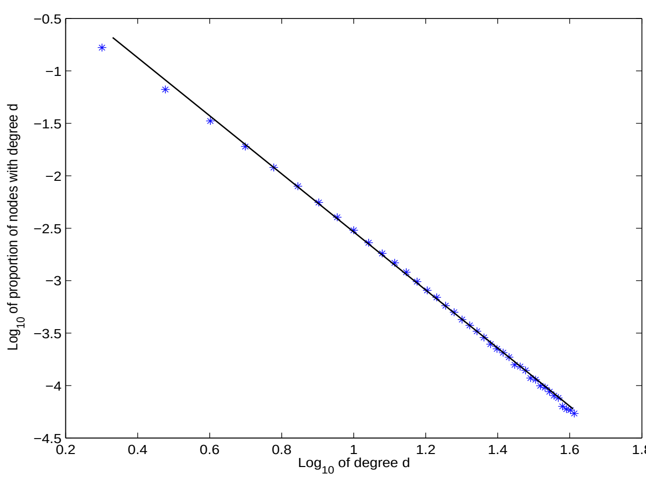
Preferential Attachment



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Network models and temporal evolution

- For most of the existing models it is assumed that
  - number of edges grows linearly with the number of nodes
  - the diameter grows at rate logn, or loglogn
- What about real graphs?
  - Leskovec, Kleinberg, Faloutsos 2005

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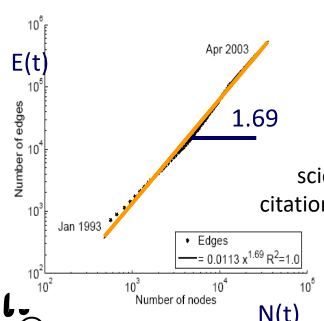
Densification laws

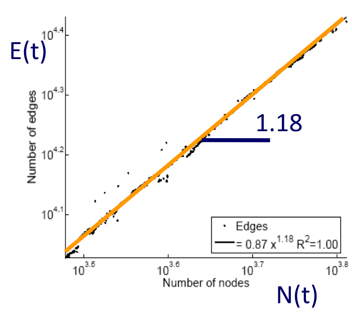
- In real-life networks the average degree increases! – networks become denser!

$$\boxed{E(t) \propto N(t)^{a}}$$

α = densification exponent



scientific citation network



Internet

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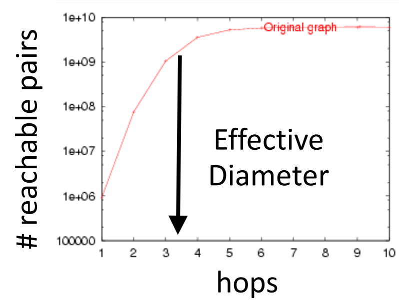
What about diameter?

- Effective diameter: the interpolated value where 90% of node pairs are reachable



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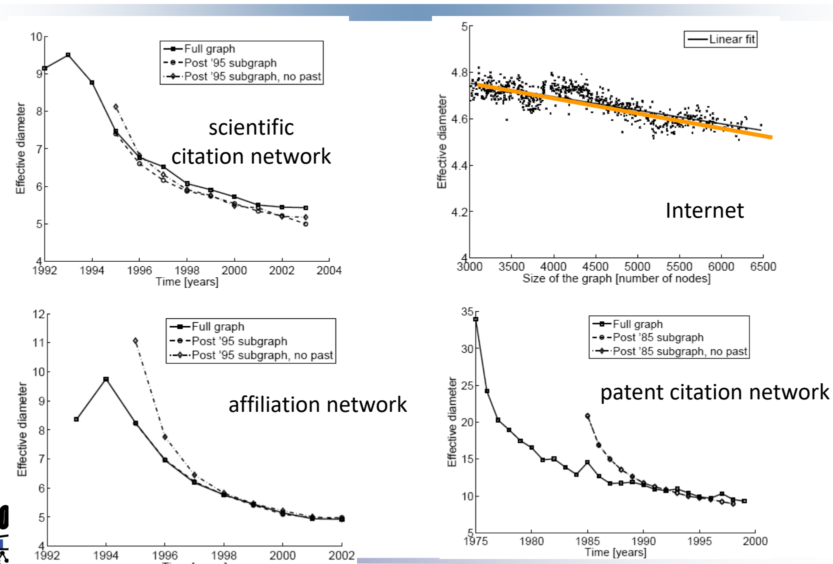
Diameter shrinks



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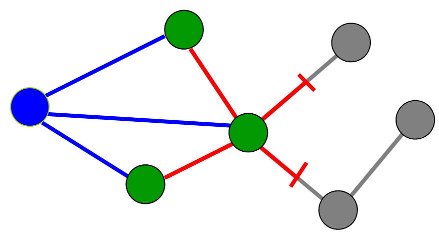
"Forest Fire" Model

- A node arrives
- Randomly chooses an "ambassador"
- Starts burning nodes (with probability p) and adds links to burned nodes
- "Fire" spreads recursively



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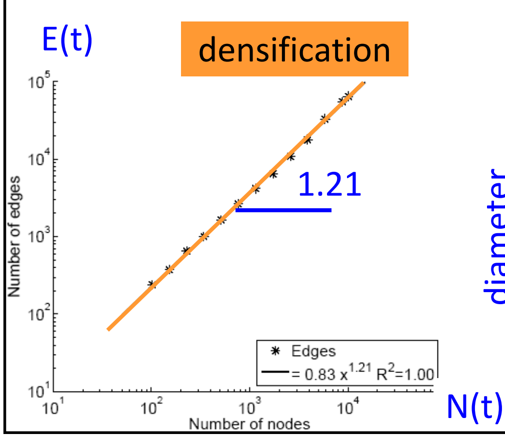
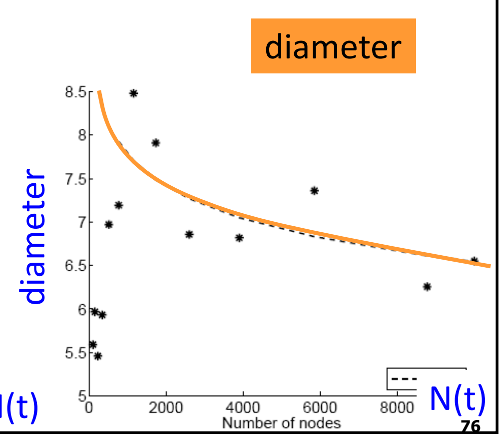
Forest Fire in Action (1)

- Forest Fire generates graphs that Densify and have Shrinking Diameter





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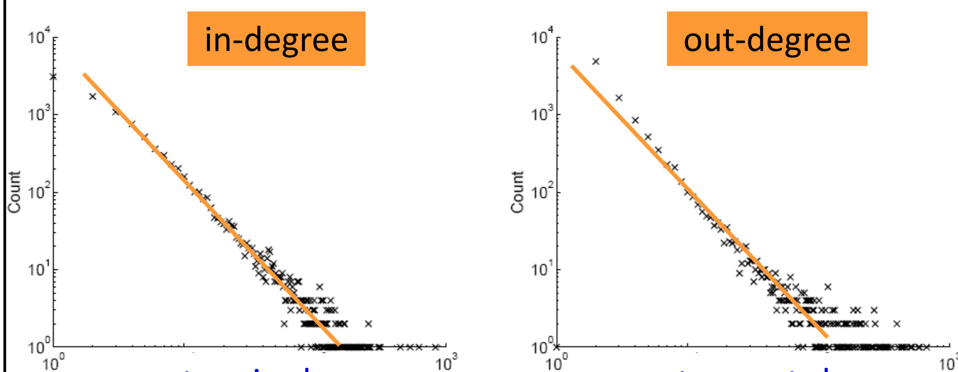
Forest Fire in Action (2)

- Forest Fire also generates graphs with heavy-tailed degree distribution



count vs. in-degree

count vs. out-degree

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Forest Fire model – Justification

- Densification Power Law:
  - Similar to Community Guided Attachment
  - The probability of linking decays exponentially with the distance – Densification Power Law
- Power law out-degrees:
  - From time to time we get large fires
- Power law in-degrees:
  - The fire is more likely to reach hubs
- Communities:
  - Newcomer copies neighbors' links
- Shrinking diameter

What is Flowing?

- Things that can flow
  - Ideas or Beliefs
  - Money or Resources
  - New Technology
  - Disease
- Each has different flow properties because
  - Retention
  - Acceptance
  - Resistance

Propagation models

- Epidemics
  - How do epidemic diseases propagate through society?
  - One of the major reasons that people started studying social networks in the community
- Consumer's society
  - How do products propagate and innovations get accepted ?
  - Early reason for studying online social networks
- Fads
  - How do ideas and beliefs diffuse?
  - One of the major reasons that people started studying social networks in the workplace

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ELEMENTS OF THE DIFFUSION OF INNOVATIONS

1. The rate of diffusion is influenced by the perceived characteristics of the innovation such as relative advantage, compatibility, observability, trialability and complexity, radicalness, and cost.
2. Diffusion occurs over time such that the rate of adoption often yields a cumulative adoption S-shaped pattern.
3. Individuals can be classified as early or late adopters.
4. Individuals pass through stages during the adoption process typically classified as (1) knowledge, (2) persuasion, (3) decision, (4) implementation or trial, and (5) confirmation.

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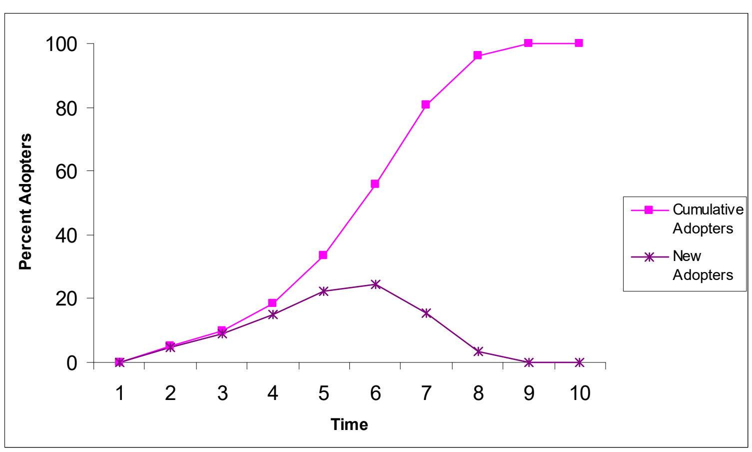
Hypothetical Cumulative and Incidence Adoption Curves for Diffusion Homogenous Mixing



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Critical Levels

- Tipping points
- Macro vs. micro tipping points, critical mass vs. thresholds
- Most CM/threshold models were not explicitly social network explanations

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Statistical Process Control (SPC)

- Change detection can be based on SPC
- What is Statistical Process Control?
  - Used in manufacturing to maintain quality control
  - Monitors a process to detect potential changes
  - Calculates a statistic from observed measurements of a process and compares it to a decision interval
  - If the statistic exceeds the decision interval, it is said to "signal", that a potential change may have occurred
  - A quality engineer will then begin to search for the specific cause of change

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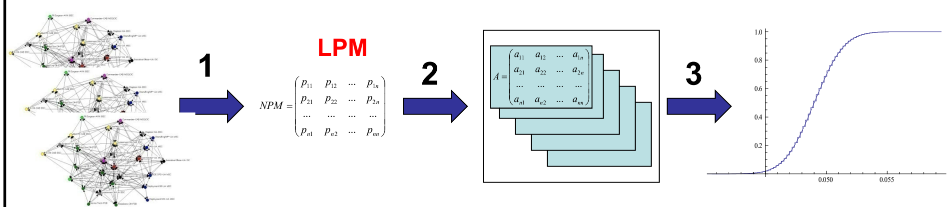
Statistical Models of Networks
Link Probability Model (LPM) for Stability

- LPM is a model for a network in *Stability*
- The probability that an email is sent from *i* to *j* within some period of time *t* is: $$p = \int_0^t f_{ij}(x \mid \theta_{ij})\,dx$$
  - (p, as a function of t, is a CDF: f is the PDF that best fits cell ij in an NPM)
- LPM can be used to simulate stable longitudinal networks



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Statistical Models of Networks
Link Probability Model (LPM) for Stability

LPM simulated networks are compared to empirical networks and are shown to represent the network well.

M	*8*		*N*	*60000*			
e_mean	**e_stdev**		**s_mean**	**s_stdev**		**t-val**	**p**
409.2857	38.5604		358.0939	12.77466		3.754923	0.00
365.8571	18.2978		320.0974	12.7394		7.073195	0.00
365.8571	29.04266		320.1638	12.79331		4.449958	0.00
377.8571	38.24669		330.6744	12.77289		3.489244	0.00
375.2857	36.10039		328.3765	12.79551		3.675254	0.00
349.8571	38.15944		306.0783	12.7845		3.244918	0.00
373.8571	48.45076		327.0728	12.82622		2.731135	0.01
362.4286	55.63529		317.1509	12.77754		2.301849	0.02

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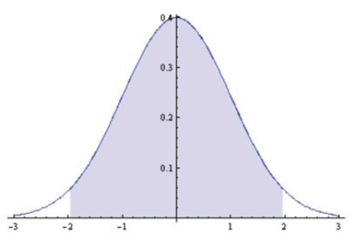
Probability Background

- Consider a normal distribution with μ=0 and σ=1.
- 95% of the time, observations are between ±1.9597
- When an observation occurs in the tail, we don't believe it and think that something unusual might be going on.



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